Chamchuri Journal of Mathematics
Volume 7(2015), 1–15
http://www.math.sc.chula.ac.th/cjm

Anti-codes on solid burst of length b of anti-weight t

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Received 7 November 2014
Revised 2 March 2015
Accepted 2 April 2015

Abstract: This paper considers a new kind of error which will be termed as 'solid burst of length b of anti-weight t'. Lower and upper bounds on the number of parity checks required for the existence of anti-codes that detect solid burst of length b or less of anti-weight t or more are obtained. This is followed by an example of such anti-codes. The paper also deals with anti-codes capable of detecting and simultaneously correcting such errors. Then the maximum anti-weight of such errors in the space of n-tuples is discussed. Further, the paper obtains an upper bound on the number of parity checks required for the existence of anti-codes that detect solid burst of length b or less of anti-weight t or more, together with e or less random errors of anti-weight t or more ($e < b$).

Keywords: Parity check matrix, syndrome, standard array, coset, solid burst error.

2010 Mathematics Subject Classification: 94B05, 94B20, 94B65

1 Introduction

The concept of anti-weight and anti-metric has been recently introduced by Jain [8] and is found suitable for channels causing errors near the end of the code words. Jain observed that some systems use to get stuck up at some position and start causing errors after that position. In view of this, in such systems care should be taken only those components that comes after that position, not all the components of the codewords are required to check. The anti-weight of error vectors determines the beginning position of the faulty area and anti-codes are developed to encounter those errors.

Jain [8] has defined anti-weight, anti-metric, anti-code and standard anti-array as follows: Let $GF(q)$ be a finite field with q elements where q is prime or power of prime. Let F be the set of all n-tuples over $GF(q)$. Then F is a vector space over $GF(q)$.

Definition 1.1. The anti-weight $T_w(v)$ of a vector $v = (v_1, v_2, ..., v_n) \in F$ is defined as $T_w(v) = min\{i | v_i \neq 0, 1 \leq i \leq n\} - 1$.

Definition 1.2. The anti-distance $T_d : F \times F \rightarrow \{0, 1, 2, ..., n\}$ is defined as $T_d(x, y) = T_w(x - y)$.

Note. The functions T_w and T_d are called T-anti-weight and T-anti-metric respectively and will be defined by the same notation T.

Definition 1.3. A T-anti-code or simply an anti-code V is a k-dimensional subspace of F equipped with the T-anti-metric.

Definition 1.4. The standard anti-array for an (n, k) anti-code V is the same as the standard array used in normal coding with coset leaders being replaced by anti-coset leaders where the anti-coset leaders are vectors of maximum anti-weights in their respective cosets and farthest neighbour decoding principle will be used for decoding purpose.

There are several kinds of errors for which error detecting and error correcting codes have been constructed. The kind of errors differs from channel to channel depending upon the behaviour of channels. Solid burst errors are common in many memory systems [1, 2, 3, 10]. The definition of a solid burst may be given as follows:

Definition 1.5. A solid burst of length b is a vector with non zero entries in some b consecutive positions and zero elsewhere.

In papers [8, 9], the authors have considered the error of the types - either random errors of anti weight t or cyclic errors of order t or anti burst of length b. This paper considers another type of errors. The type of errors is such that the errors that start occurring after a certain position is of solid burst type. The systems that are equipped with anti-weight and anti-metric concept may be affected by such type of errors and so codes are needed to be constructed to counter such errors. The new type of errors is termed as solid burst of length b of anti-weight t and is defined as follows:

Definition 1.6. A solid burst of length b of anti-weight t is a vector such that the components from $(t+1)^{th}$ to $(t+b)^{th}$ positions are nonzero and the rest are zero.

For example,
(i) (000111) is a solid burst of length 3 of anti-weight 3.
(ii) (011000) is a solid burst of length 2 of anti-weight 1.
(iii) (111111) is a solid burst of length 6 of anti-weight 0.

It is quite possible that the system which is interfered by the error in the form of solid burst of length b of anti-weight t, may also be encountered with some small number of random errors of anti-weight t. Therefore, while it is all important to consider the detection/correction of solid burst of anti-weight t, but care should also be taken to handle the detection/correction of some random errors of anti-weight t. In view of this, this paper presents a study not only on bounds on the number of parity-check digits for anti-codes that detect solid burst error of length b or less of anti-weight t or more, but also on bound on the number of parity-check digits for anti-codes detecting solid burst error of length b or less of anti-weight t or more, and e or less random errors of anti-weight t or more ($e < b$).

The rate of transmission is efficient if the number of parity-check digits is as less as possible. To give the exact number of redundant/parity check digits for a given (anti) code is usually not possible. However, bounds on the number of redundant/parity check digits can be obtained. In fact, they are important in determining error correction and error detection capabilities of the (anti) codes. Hamming [7] was the first who gave the lower bound on the necessary number of parity-check digits for the codes correcting single errors. Gilbert [6] gave a more general lower bound on the number of code words in a code with fixed length and distance. After that, many researchers have obtained various lower and upper bounds.

In coding theory, it is always important to study the asymptotic form of the different bounds (e.g. Plotkin's bounds, Hamming bounds, Varshomov-Gilbert bounds). They have been widely studied in many textbooks, e.g.[11], [12]. The standard asymptotic form is to fix q, let $n \to \infty$, and try to make the transmission rate and the error-detection/correction rate both large. The well known Gilbert-Varshamov bound is found to be quite weak for small n, but its asymptotic form is very hard to beat. In this paper, we also provide the asymptotic form of a bound obtained.

The paper is organized as follows. Basic definitions, works related to our study are stated with examples in Section 1 i.e. in Introduction. In Section 2, lower and upper bounds on the number of the parity checks for an anti-code that detects solid burst error of length b or less of anti-weight t or more are obtained. This is followed by an illustration of such anti-codes. In Section 3, we obtain a bound on anti-code for simultaneous detection and correction of such errors. Then the maximum anti-weight of such errors over the space of n-tuples over $GF(q)$ is obtained. Section 4 gives an upper bound on the parity checks for an anti-code that detects solid burst error of length b or less of anti-weight t or more, and any e or less random errors of anti-weight t or more. Also the asymptotic form of the bound is provided.

2 Detection of solid burst of length b or less of anti weight t or more

We consider linear anti-codes over GF(q) that detect any solid burst of length b or less of anti-weight t or more. The patterns that needed to be detected should not be code words. In other words, we consider anti-codes that have no solid burst of length b or less of anti-weight t or more as a code word. In the following, we obtain a *lower* bound over the number of parity-check digits required for such an anti-code. The proof is similar to the proof of the Theorem 4.13, Peterson and Weldon [14].

Theorem 2.1. *The number of parity check digits for an (n,k) linear anti-code over GF(q) that detects any solid burst of length b or less of anti-weight t or more $(b+t \le n)$ is at least* $\log_q(1+b)$.

Proof. The proof is based on the fact that no detectable error vector can be a code word. Let V be an (n,k) linear anti-code over GF(q) and $\mathcal{X}$ be a set of all those vectors such that some fixed non-zero component are in the $(t+1)^{th}$ to $(t+i)^{th}$ positions consecutively, where $1 \le i \le b$.

We claim that any two vectors of the set $\mathcal{X}$ can not belong to the same coset of the standard anti-array; otherwise a code word shall be expressible as a sum or difference of two error vectors. If possible, we assume the contrary that there is a pair; say x_1, x_2 in $\mathcal{X}$ belonging to the same coset of the standard anti-array. Then their difference $x_1 - x_2$ must be a code vector. But $x_1 - x_2$ is a vector all of whose non zero components are within the $(t+1)^{th}$ to $(t+b)^{th}$ positions consecutively. This means $x_1 - x_2$ is an error vector and is a code vector. This is not possible. Thus, all the vectors in $\mathcal{X}$ must belong to distinct cosets of the standard anti-array. The number of such vectors over GF(q), including the vector of all zero, is clearly $1+b$. Since the maximum available number of cosets is q^{n-k}, we have

$$q^{n-k} \geq 1 + b. \tag{1}$$

This proves the theorem. □

Remark 2.2. This result coincides with Theorem 1, Das [4] when burst of length b or less are considered over the whole code length.

Remark 2.3. It may be noted that the result of Theorem 2.1 is free from other parameters of the anti-code. So, the result is applicable for anti-codes of any feasible length n and anti-weight t.

Example 2.4. By taking $t = 4, b = 3, n = 7, q = 2$, the inequality (1) gives rise to a (7, 5) binary anti-code with the parity matrix H,

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

The null space of this matrix can be used to detect all solid bursts of length 3 or less of anti-weight 4 or more. It may be verified from error pattern-syndromes Table 2.1 that the syndromes of all solid bursts of length 3 or less of anti-weight 4 or more are non zero.

Table 2.1

Error pattern - syndromes

Error-patterns	Syndromes
Solid bursts of length 1 of anti weight 4 or more	
0000100	10
0000010	01
0000001	10
Solid bursts of length 2 of anti weight 4 or more	
0000110	11
0000011	11
Solid bursts of length 3 of anti weight 4 or more	
0000111	11

Remark 2.5. The lower bound obtained in Theorem 2.1 is only a necessary condition for the existence of anti-code over $GF(q)$ that detects any solid burst of length b or less of anti-weight t or more. The bound in Theorem 2.1 only shows that if such an anti-code exists, it always satisfies the bound. However, it is not a sufficient (upper) bound. For example, by taking $t = 3, b = 4, n = 7, q = 3$, the inequality (1) gives the possibility of $(7,5)$ ternary anti-code. But there does not exist any $(7,5)$ ternary anti-code that detects any solid burst of length 4 or less of anti-weight 3 or more.

Now the following theorem gives an *upper* bound on the number of check digits required for the construction of an anti-code considered in Theorem 2.1. This bound assures the existence of an anti-code that can detect all solid bursts of length b or less of anti-weight t or more. The proof is based on the well known technique used in Varshomov-Gilbert Sacks bound by constructing a parity check matrix for such a code (refer Sacks [16], also Theorem 4.7 Peterson and Weldon [14]).

Theorem 2.6. *There exists an (n,k) linear anti-code over $GF(q)$ that detects any solid burst of length b or less of anti-weight t or more $(b+t \leq n)$ provided that*

$$n - k \geq loq_q\left\{1 + \sum_{i=0}^{b-1}(q-1)^i\right\}. \quad (2)$$

Proof. The theorem is proved by constructing an appropriate $(n-k)\times n$ parity-check matrix H for the existence of the anti-code. The requisite parity-check matrix H shall be constructed as follows:
Select any non-zero $(n-k)$-tuples as the first t columns $h_1, h_2, ..., h_t$. After selecting $n-t-1$ columns $h_{t+1}, h_{t+2}, ..., h_{n-1}$ appropriately, we lay down the condition to add n^{th} column h_n such that it should not be a linear sum of immediately preceding consecutive upto $b-1$ columns. In other words,

$$h_n \neq (u_1 h_{n-1} + u_2 h_{n-2} + \cdots + u_{s-2} h_{n-s+2} + u_{s-1} h_{n-s+1}), \tag{3}$$

where $u_i \in GF(q)$ are non zero coefficients and $s \leq b$.

This condition ensures the existence of the anti-code detecting any solid burst of length b or less of anti-weight t or more. The number of ways in which such coefficients u_i on R.H.S. of (3) may be chosen is given by

$$\sum_{i=0}^{b-1} (q-1)^i.$$

At worst, all these linear combinations might yield distinct sums. Thus, a column h_n can be added to H provided that

$$q^{n-k} > \sum_{i=0}^{b-1} (q-1)^i \tag{4}$$

or,

$$q^{n-k} \geq 1 + \sum_{i=0}^{b-1} (q-1)^i$$

or,

$$n-k \geq log_q \left\{ 1 + \sum_{i=0}^{b-1} (q-1)^i \right\}.$$

□

Remark 2.7. This result coincides with Theorem 2, Das [4] when solid burst of length b or less are considered over the whole code length.

Remark 2.8. This result is also free from other parameters of the anti-code. So, the result is applicable for anti-codes of any feasible length n and anti-weight t.

Remark 2.9. For $q = 2$, the bounds obtained in Theorem 2.1 and Theorem 2.6 coincide i.e., the lower and upper bounds on the number of parity check digits for an (n, k) linear anti-code over $GF(2)$ that detects any solid burst of length b or less of anti-weight t or more is same and is given by $loq_2(1 + b)$.

Example 2.10. Consider a (7, 5) binary anti-code with the 2×7 matrix H which has been constructed by the synthesis procedure given in the proof of Theorem 2.6 by taking $t = 4, b = 3, n = 7$.

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

The null space of this matrix detects all solid bursts of length 3 or less of anti-weight 4 or more. Because the syndromes of all solid bursts of length 3 or less of anti-weight 4 or more are non zero (refer Table 2.1).

3 Simultaneous detection and correction of solid burst of length b or less of anti weight t or more

This section determines extended Reiger's bound (refer [17] ; also Theorem 4.15, Peterson and Weldon [14]) for simultaneous detection and correction of solid burst of length b or less of anti-weight t or more. The following theorem gives a bound on the number of parity-check digits for a linear anti-code that simultaneously detects and corrects such errors.

Theorem 3.1. *The number of parity check symbols in an (n, k) linear anti-code over GF(q) that corrects any solid burst of length b or less of anti-weight t or more must have at least* $\log_q(1 + 2b)$.

Further, if the anti-code corrects all solid bursts of length b or less of anti-weight t or more, and simultaneously detects any solid burst of length d or less of anti-weight t or more $(b < d)$, then the number of parity-check digits of the anti-code is at least $\log_q(1 + b + d)$.

Proof. For the first part, consider a vector that has the form of a solid burst of length $2b$ or less of anti-weight t or more. The vector can be expressible as a sum or difference of two vectors, each of which is a solid burst of length b or less of anti-weight t or more. These component vectors must belong to different cosets of the standard anti-array, because both such errors are correctable errors. Accordingly, such a vector viz. a solid burst of length b or less of anti-weight t or more can not be a code vector. In view of Theorem 2.1, the number of parity check digits, such an anti-code must have, is at least $\log_q(1+2b)$.

Further, for the second part, consider a vector which has the form of a solid burst of length $(b+d)$ or less of anti-weight t or more. Such a vector is expressible as a sum or difference of two vectors, one of which has the form of a solid burst of length b or less of anti-weight t or more and the other is a solid burst of length d or less of anti-weight t or more. Both such component vectors, one being a detectable error and the other being a correctable error, can not belong to the same coset of the standard anti-array. Therefore, such a vector can not be a code vector, i.e., a vector which is a solid burst of length $(b+d)$ or less of anti-weight t or more can not be a code vector. Hence, by Theorem 2.1, the number of parity check digits that anti-code must have is at least $\log_q(1+b+d)$. □

In coding theory, an important criterion is to look for minimum weight and structure of weight in a group of vectors. For anti-code, we need to look for maximum anti-weight structure of error vectors. The following theorem gives the maximum anti-weight of solid burst of length b or less of anti-weight t or more. The theorem is equivalent to Plotkin bound [15], also Theorem 4.1, Peterson and Weldon [14]).

Theorem 3.2. *The maximum anti-weight of a solid burst of length b or less of anti-weight t or more in the space of n-tuples over $GF(q)$ is at least*

$$\frac{\sum_{l=1}^{b} \dfrac{(n-l)^2-t^2}{2} \times (q-1)^l}{\sum_{i=1}^{b} (n-t-i+1)(q-1)^i}.$$

Proof. We first count the total anti-weight of all solid bursts of length l $(1 \le l \le b)$ of anti-weight t or more. This is given by

$$(q-1)^l \Big\{ t+(t+1)+(t+2)+\cdots+(n-l) \Big\} = (q-1)^l \times \frac{(n-l)^2-t^2}{2}.$$

Therefore, the total anti-weight of all solid bursts of length b or less of anti-weight

t or more is given by

$$\sum_{l=1}^{b} \frac{(n-l)^2 - t^2}{2} \times (q-1)^l.$$

Also, the number of solid burst of length b or less of anti-weight t or more in the space of n-tuples over $GF(q)$ is

$$\sum_{i=1}^{b} (n-t-i+1)(q-1)^i.$$

Since the maximum anti-weight element can have at least the average anti-weight, a lower bound on the maximum anti-weight of solid burst of length b or less of anti-weight t or more is given by

$$\frac{\sum_{l=1}^{b} \frac{(n-l)^2 - t^2}{2} \times (q-1)^l}{\sum_{i=1}^{b} (n-t-i+1)(q-1)^i}.$$

Hence the theorem is proved. □

4 Detection of solid burst of length b or less of anti weight t or more and random errors

In this section we study the sufficient condition (upper bound) for the detection of solid burst of length b or less of anti-weight t or more, and e or less random errors of anti-weight t or more ($e < b$). The proof of the following theorem is analogous to that of Theorem 2.6. Further, the asymptotic form of the bound obtained is provided. The study is parallel to the works done by Dass and Muttoo [5], Muttoo and Tyagi [13] where they have considered the codes detecting closed loop burst errors and random errors.

Theorem 4.1. *There exists an (n,k) linear anti-code over $GF(q)$ that detects any solid burst of length b or less of anti-weight t or more, and e or less random errors of anti-weight t or more ($e < b$) provided that*

$$q^{n-k} > \sum_{i=0}^{e-1} \binom{n-t-1}{i} (q-1)^i + \sum_{i=e}^{b-1} (q-1)^i. \tag{5}$$

Proof. The theorem is also proved by constructing an appropriate $(n-k)\times n$ parity-check matrix H for the existence of the anti-code. This requisite parity-check matrix H is constructed as follows:

Select any non-zero $(n-k)$-tuples as the first t columns $h_1, h_2, ..., h_t$. After selecting $n-t-1$ columns $h_{t+1}, h_{t+2}, ..., h_{n-1}$ appropriately, we lay down the conditions to add n^{th} column h_n as follows:

Case (I). Since the anti-code detects any e or less random errors of anti-weight t or more, h_n should not be a linear combination of previous any $e-1$ or less columns among the immediately preceding $n-t-1$ columns, i.e.,

$$h_n \neq (u_1 h_{n-1} + u_2 h_{n-2} + \cdots + u_{n-t-2} h_{t+2} + u_{n-t-1} h_{t+1}), \tag{6}$$

where $u_i \in GF(q)$ are any $e-1$ or less non zero coefficients.

This condition ensures that the anti-code detects any e or less random errors of anti-weight t or more. The number of ways in which such coefficients u_i out of $n-t-1$ coefficients on R.H.S. of (6) may be chosen is given by

$$\sum_{i=0}^{e-1} \binom{n-t-1}{i} (q-1)^i.$$

Case (II). Since the anti-code detects any solid burst b or less of anti-weight t or more, h_n should not be a linear combination of immediately previous any e or more (but less than equal to $b-1$) consecutive columns, i.e.,

$$h_n \neq (u_1 h_{n-1} + u_2 h_{n-2} + \cdots + u_{s-1} h_{n-s+1} + u_s h_{n-s}), \tag{7}$$

where $e \leq s \leq b-1$ and $u_i \in GF(q)$ are non zero coefficients.

This condition ensures that the anti-code detects any solid burst of length b or less (but greater than e) of anti-weight t or more. The number of ways in which such coefficients u_i on R.H.S. of (7) may be chosen is given by

$$\sum_{i=e}^{b-1} (q-1)^i.$$

Therefore, the number of possible linear combinations of coefficients for the case (I) and case (II), including the vector of all zeros, is given by

$$\sum_{i=0}^{e-1}\binom{n-t-1}{i}(q-1)^i+\sum_{i=e}^{b-1}(q-1)^i.$$

At worst, all these linear combinations might yield distinct sums. Thus, a column h_n can be added to H provided that

$$q^{n-k}>\sum_{i=0}^{e-1}\binom{n-t-1}{i}(q-1)^i+\sum_{i=e}^{b-1}(q-1)^i. \tag{8}$$

□

Alternative Form

If n is the largest positive integer for which the inequality (8) is true, then replacing n by $n+1$, the inequality (8) gets reversed and the inequality becomes

$$q^{n-k}\leq\sum_{i=0}^{e-1}\binom{n-t}{i}(q-1)^i+\sum_{i=e}^{b-1}(q-1)^i. \tag{9}$$

Asymptotic Form

We deduce the asymptotic form of the above inequality (9) over GF(2). By taking $q=2$, the inequality (9) becomes

$$2^{n-k}\leq\sum_{i=0}^{e-1}\binom{n-t}{i}+(b-e)$$

or,

$$2^{n-k}\leq\sum_{i=(\frac{n-t-e+1}{n-t})(n-t)}^{n-t}\binom{n-t}{i}+(b-e). \tag{10}$$

From the Chernov Bound,

$$\sum_{i=\alpha n}^{n}\binom{n}{i}\leq\alpha^{-\alpha n}\beta^{-\beta n},\quad \beta=1-\alpha,\quad \alpha>\frac{1}{2},$$

we can deduce the inequality (10) as follows:

$$2^{n-k}\leq\left(\frac{n-t-e+1}{n-t}\right)^{-(n-t-e+1)}\left(\frac{e-1}{n-t}\right)^{-(e-1)}+(b-e), \tag{11}$$

where $n > t + 2(e-1)$. We know the binary entropy function is given by

$$H(p) = -p\log_2 p - (1-p)\log_2(1-p).$$

Therefore, the inequality (11) reduces to

$$2^{n-k} \leq 2^{(n-t)H(\frac{n-t-e+1}{n-t})} + b - e.$$

Remark 4.2. We have restricted the asymptotic form to $q = 2$, because for non binary case, we can not apply Chernov Bound in (9), also the entropy function is given by

$$H_q(p) = -p\log_q p - (1-p)\log_q(1-p) + p\log_q(q-1).$$

Example 4.3. For a $(10, 6)$ linear anti-code over $GF(2)$, we construct the following 4×10 parity check matrix H, according to the synthesis procedure given in the proof of Theorem 4.1 by taking $q = 2$ $e = 2$, $b = 4$ and $t = 2$.

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

The null space of this matrix can be used to detect all solid bursts of length 4 or less of anti-weight 2 or more, and 2 or less random errors of anti-weight 2 or more. It can easily be verified from error pattern-syndromes table that the syndromes of all solid bursts of length 4 or less of anti-weight 2 or more, and 2 or less random errors of anti-weight 2 or more are non zero, showing thereby that the anti-code that is the null space of this matrix can detect all such errors.

Acknowledgement

The author is thankful to referees for their careful reading of the manuscript and for their valuable suggestions which help to improve the paper.

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