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Rayleigh-Benard Convection in a Dielectric Liquid : Imposed Time-Periodic Boundary Temperatures

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Abstract: We discuss the thermal instability in a layer of dielectric fluid when the boundaries of the layer are subjected to synchronous/asynchronous time-periodic temperatures. Only infinitesimal disturbances are considered. Perturbation solution in powers of the amplitude of the applied temperature field is obtained. In the case when the Imposed Time-periodic Boundary Temperatures (ITBT) at the two walls are synchronized then for moderate values of frequency the role of the electric Rayleigh Number in inducing subcritical instabilities is delineated. A similar role is shown to be played by the Prandtl number. The dielectric parameters and Prandtl number have the opposite effect at large frequencies. The system is most stable when the ITBT is asynchronous. The problem has relevance in many dielectric fluid applications wherein regulation of thermal convection is called for.

1 Introduction

One of the effective mechanisms of controlling convection is through the maintenance of a non-uniform temperature gradient which is only space-dependent. However, in many practical situations non-uniform temperature gradients find their origin in transient heating or cooling at the boundaries, hence warranting the use of a basic temperature profile which is a function of both position and time. Venezian [15] investigated the stability of a horizontal layer of a viscous fluid heated from below when, in addition to a steady temperature difference between the surfaces of the layer, a time-dependent sinusoidal perturbation is applied to the wall temperatures. Subsequently, it was shown by Yih and Li [13] that time-periodic modulation of the wall temperatures has a destabilizing effect on the onset of convection over a wide range of frequencies of modulation although such a modulation is stabilizing for low frequencies. The critical Rayleigh number (corresponding to onset of convection) in these problems depends on the frequency of the imposed temperature modulation and the study suggests that it is possible to hasten or delay the onset of instability by adjusting this modulation. Studies have also been made on the effect of synchronous/asynchronous ITBT on convection ([11], [12]) in suspensions.

The application of a strong electric field in a poorly conducting fluid can induce bulk motions. This phenomenon known as electroconvection or electrohydrodynamics is gaining importance due to the technological stimulus of designing more efficient heat exchangers as required for jet engines [6]. Boiling of dielectric fluids was proposed as a promising cooling mechanism for future microelectronic chips [2, 3]. EHD convection is very attractive in applications to new fluid devices such as in a dielectric fluid motor [5]. Since magnetic fields and switching circuits are not required the dielectric fluid motor enhances size reduction and hence is an attractive source of mechanical energy in a micro machine. Convective heat transfer through polarized dielectric liquids was studied by P. J. Stiles [7, 8].

The problem of control of convection is of relevance and interest in innumerable dielectric fluid applications [4, 12, 13, 14] and is also mathematically quite challenging. It is with this motivation we study the problem of the ITBT-means of regulating convection. We determine the onset of convection for a dielectric fluid layer heated from below, when, in addition to a fixed temperature difference between the walls, an additional time-periodic perturbation is applied to the wall temperatures.

2 Mathematical Formulation and Solution

We consider an infinite horizontal layer of a dielectric fluid of thickness h.
We choose a cartesian co-ordinate system x, y, z in which z is measured at right angles to the boundaries and the origin is on the lower boundary. The lower plane surface is at $z = 0$ and the upper one is at $z = h$. The lower surface is grounded

and the upper surface is kept at a high alternating (60 Hz) potential. The wall temperatures are externally imposed and are taken as

$$T_0 + \frac{1}{2}\Delta T\,[1 + \varepsilon \cos \omega\, t] \text{ at } z = 0 \text{ and } T_0 - \frac{1}{2}\Delta T\,[1 - \varepsilon \cos (\omega\, t + \Phi)] \text{ at } z = h \quad (1)$$

where T_0 is a reference temperature, ΔT is the temperature difference between the two walls in the unmodulated case, ε is the amplitude of the thermal modulation, ω is the frequency and Φ is the phase.

We adopt the Boussinesq approximation and for small departures from T_0, the density ρ, as a function of temperature T, is given by

$$\rho = \rho_0\,[1 - \alpha\,(T - T_0)]\,. \quad (2)$$

The Navier-Stokes equations describing flow in an incompressible dielectric fluid are

$$\nabla.\,\vec{q} = 0, \quad (3)$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\,\vec{q}\right] = -\nabla p + \rho \vec{g} + \eta \nabla^2 \vec{q} + \vec{P}.\nabla \vec{E}, \quad (4)$$

$$\frac{\partial T}{\partial t} + \vec{q}.\nabla T = K_c \nabla^2 T, \quad (5)$$

$$\varepsilon = \varepsilon_0\,[1 - e\,(T - T_0)]\,, \quad (6)$$

$$\nabla.\,\left(\varepsilon_0 \vec{E} + \vec{P}\right) = 0, \quad (7)$$

$$\nabla \times \vec{E} = 0 \quad (8)$$

where α and e are usually positive.

In the above equations, $\vec{q}$ is the velocity, T is the temperature, p is the pressure, η is the shear kinematic viscosity co-efficient, α is the co-efficient of thermal expansion, ρ is the density, ρ_0 is the density of the fluid at temperature $T = T_0$, K_c is thermal diffusivity.

Equation (8) allows us to express the irrotational electric field $\vec{E}$ as

$$\vec{E} = E_0 \left[1 + \beta\, z \left(\frac{\partial\,(\ln \varepsilon_r)}{\partial T}\right)\right] \hat{k} - \nabla \phi, \quad (9)$$

and the polarization field $\vec{P}$ can be expressed as

$$\vec{P} = \varepsilon_0 E_0 \left[\varepsilon_r - 1 - \left(\frac{\partial \ (\ln \varepsilon_r)}{\partial T} \right) (\beta z - T) \right] \hat{k} - (\varepsilon_r - 1)\, \varepsilon_0 \nabla \phi \tag{10}$$

where ϕ is the perturbation to the electric scalar potential due to convection, ε_r and ε_0 are the relative permittivity and permittivity of free space respectively and β is the temperature gradient.

We now study the condition for onset of convection in the aforementioned dielectric fluid layer. In the undisturbed state, the equations (2) - (6) yield

$$E_b = -\frac{\phi_1 e\,(T_0 - T_b)\, h}{\log\left(1 + e\,(T_0 - T_b)\right)}, \tag{11}$$

$$-\frac{\partial p_b}{\partial z} = \rho_b g - P_b \frac{\partial E_b}{\partial z}, \tag{12}$$

$$\frac{\partial T_b}{\partial t} = K_c \frac{\partial^2 T_b}{\partial z^2}, \tag{13}$$

$$\rho_b = \rho_0 \left[1 - \alpha\,(T_b - T_0) \right], \tag{14}$$

$$\varepsilon_b = \varepsilon_0 \left[1 - e\,(T_b - T_0) \right] \tag{15}$$

where the subscript b denotes quantities in the basic state.

Following Venezian [15], the solution of (13) satisfying the thermal boundary conditions (1) is

$$T_b = T_0 + \frac{\Delta T}{2h}(h - 2z) + \varepsilon Re \left\{ \left[a\,(\lambda)\, e^{\frac{\lambda z}{h}} + a\,(-\lambda)\, e^{\frac{-\lambda z}{h}} \right] e^{-i\omega t} \right\}, \tag{16}$$

where

$$\lambda = (1 - i) \left(\frac{\omega h^2}{2K_c} \right)^{\frac{1}{2}}, a\,(\lambda) = \frac{\Delta T}{2} \left[\frac{e^{-i\Phi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}} \right], \tag{17}$$

and Re stands for the real part.

2.1 Linear Stability Analysis

Let the basic state be slightly perturbed by an infinitesimal perturbation. Linearizing the equations governing the infinitesimal perturbations, operating curl twice on the resulting momentum equation and writing in dimensionless form by setting

$$(x^*, y^*, z^*) = \left(\frac{x}{h}, \frac{y}{h}, \frac{z}{h}\right), \vec{q}^* = \frac{\vec{q}'}{(K_c/h)}, \theta^* = \frac{\theta}{\Delta T},$$

$$\phi^* = \frac{\phi'}{E_0 \beta \, h^2 \left(\frac{d(\ln \varepsilon_0)}{dT}\right)}, t^* = \frac{t}{(h^2/K_c)}. \tag{18}$$

We get

$$\frac{\partial \left(\nabla^2 w\right)}{\partial t} - \Pr\left(R + L\right) \nabla_1^2 \theta + \Pr L \frac{\partial \left(\nabla_1^2 \phi\right)}{\partial z} = \Pr \nabla^4 w, \tag{19}$$

$$\frac{\partial \theta}{\partial t} + w \frac{\partial T_b}{\partial z} = \nabla^2 \theta + w, \tag{20}$$

$$\nabla^2 \phi - \frac{\partial \theta}{\partial z} = 0, \tag{21}$$

where the asterisks have been dropped for simplicity and the non-dimensional parameters are

$$R = \frac{\rho_0 \alpha g \Delta T d^3}{\eta K_c} \qquad \text{(Rayleigh number)},$$

$$L = \left(\frac{d\varepsilon}{dT}\right)^2 \frac{E^2 \left(\Delta T\right)^2 d^2}{\eta \varepsilon K_c} \qquad \text{(Electric Rayleigh number)},$$

$$\Pr = \frac{\nu}{K_c} \qquad \text{(Prandtl number)}.$$

In equation (20), $\frac{\partial T_b}{\partial z}$ is given by

$$\frac{\partial T_b}{\partial z} = -1 + \varepsilon \, f, \tag{22}$$

where $f = Re\left\{\left(A\left(\lambda\right) \, e^{\lambda z} + A\left(-\lambda\right) e^{-\lambda z}\right) \, e^{-i\omega t}\right\}$ and $A\left(\lambda\right) = \frac{\lambda}{2}\left[\frac{e^{-i\Phi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right]$.

Equations (19)-(21) are solved subject to the following conditions appropriate for stress-free, isothermal boundaries:

$$W = \frac{\partial^2 W}{\partial z^2} = \theta = \frac{\partial \phi}{\partial z} = 0 \text{ at } z = 0, 1. \tag{23}$$

This simple boundary condition (23) facilitates an analytical solution as obtained in the present problem.

Combining equations (19)-(21) we obtain the following equation for the vertical component of velocity W in the form

$$\left[\nabla^4\left(\nabla^2-\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t}\right)\left(\nabla^2-\frac{\partial}{\partial t}\right)+L\left(\frac{\partial T_b}{\partial z}-1\right)\nabla_1^4\right]W = R\nabla^2\left(\frac{\partial T_b}{\partial z}-1\right)\nabla_1^2 W\,. \tag{24}$$

In dimensionless form, the velocity boundary conditions are (see Chandrashekar[1])

$$W=\frac{\partial^2 W}{\partial z^2}=\frac{\partial^4 W}{\partial z^4}=\frac{\partial^6 W}{\partial z^6}=0 \text{ at } z=0,1, \tag{25}$$

where the sixth order condition has been derived from the governing equations.

2.2 Stability Analysis

Let us now seek the eigenfunctions W and the eigenvalues R of equation (2.1) for the basic temperature distribution (22) that departs from the linear profile $\frac{\partial T_b}{\partial z}=-1$ by quantities of order ε. Thus, the eigenvalues of the present problem differ from those of ordinary Benard convection by quantities of order ε. We seek a solution of (2.1) in the form

$$\begin{aligned} W &= W_0+\varepsilon W_1+\varepsilon^2 W_2+\ldots, \\ R &= R_0+\varepsilon R_1+\varepsilon^2 R_2+\ldots, \end{aligned} \tag{26}$$

where R_0 is the critical Rayleigh number for the unmodulated convection in dielectric fluids. Substituting equation (26) into equation (2.1) and equating powers of ε, we obtain the following system of equations :

$$L_1 W_0 = 0, \tag{27}$$

$$L_1 W_1 = \left[\left(R_1\nabla^2 - \mathrm{R}_0 f\right)\ \nabla^2 - Lf\nabla_1^2\right]\nabla_1^2 W_0, \tag{28}$$

$$L_1 W_2 = -R_0 f\nabla^2\left[\nabla_1^2 W_1\right]\ -\mathrm{R}_1\nabla^2\left[\nabla_1^2\left(fW_0-2W_1\right)\right]\ +2\mathrm{R}_2\nabla^2\nabla_1^2 W_0, \tag{29}$$

where

$$L_1 = \nabla^4\left(\nabla^2 - \frac{1}{\Pr}\frac{\partial}{\partial t}\right)\left(\nabla^2 - \frac{\partial}{\partial t}\right) - \nabla_1^2\left(L\nabla_1^2 + \nabla^2 R_0\right). \tag{30}$$

The function W_0 is the solution of the unmodulated problem in dielectric fluids (Stiles et al [7, 8]). The marginally stable solution for that problem is

$$W_0 = \exp\left\{i\left(k_x x + k_y y\right)\right\}\sin\pi\, z, \tag{31}$$

corresponding to the lowest mode of convection with the electric Rayleigh number given by (see Stiles et al [7, 8]) :

$$L = \frac{\left(\pi^2 + a^2\right)^4 - R_0 a^2\left(\pi^2 + a^2\right)}{a^4}. \tag{32}$$

Equation (28) on using equation (31) becomes

$$L_1 W_1 = \left\{\ 2R_1 a^2\left[\pi^2 + a^2\right]\ \sin\pi\, z - R_0 f a^2\left[\pi^2 + a^2\right]\ \ - Lfa^4\right\}\ \sin\pi\, z. \tag{33}$$

If the above equation is to have a solution, then the right hand side must be orthogonal to the null space of the operator L_1. This implies that the time-independent part of the right hand side of (33) must be orthogonal to $\sin\pi z$. Since f varies sinusoidally in time, the only steady term is $2R_1 a^2\left[\pi^2 + a_3^2\right]\sin\pi\, z$, so that R_1 is zero. This result could have been anticipated because changing the sign of ε merely amounts to a shift in the time origin by half a period. Since such a shift does not affect the stability problem, it follows that all the odd coefficients $R_1,\ R_3, \ldots$ in equation (26) must vanish.

To solve equation (33) we expand the right hand side in a Fourier series and obtain W_1 by inverting the operator L_1 term by term. Following Venezian [15], we arrive at the following expression for R_2.

$$R_2 = \frac{-R_0\left[\pi^2 + a^2\right] - La^2}{4\left[\pi^2 + a^2\right]}\sum_{n=1}^{\infty}\left(R_0\left(n^2\pi^2 + a^2\right)a^2 + La^4\right)\frac{\left|B_n\left(\lambda\right)\right|^2}{\left|L_1(\omega, n)\right|^2}\frac{L_1\left(\omega, n\right) + L_1^*\left(\omega, n\right)}{2}, \tag{34}$$

where the asterisk indicates complex conjugate.

2.3 Minimum Rayleigh Number for Convection

The value of R obtained by this procedure is the eigenvalue corresponding to the function W which, though oscillating, remains bounded in time and is a function of 'a', the horizontal wave number and ε the amplitude of modulation. Thus we get

$$R(a,\varepsilon) = R_0(a) + \varepsilon^2 R_2(a) + \ldots . \tag{35}$$

At $a = a_c$ we get the critical value R_c of R. Upto order ε^2, R_c is determined by evaluating R_0 and R_2 at $a = a_0$. It is only when one wishes to evaluate R_4 that a_2 must be taken into account where $a = a_2$ minimizes R_2. To evaluate the critical value of R_2 (denoted by R_{2c} one has to substitute $a = a_0$ in R_2 where a_0 is the value of 'a' at which R_0 given by equation (32) is minimum.

We now evaluate R_{2c} for three cases:

(a) Synchronous ITBT, which means that the two ITBT's are in-phase $\Phi = 0$. In this case $B_n(\lambda) = b_n$ or 0 accordingly as n is even or odd.

(b) Asynchronous ITBT, which means that the two ITBT's are out-of-phase.

In this case we consider two sub-cases:

Type (i) : There is a phase difference between the two ITBT's with $\Phi = \pi$. In this case $B_n(\lambda) = 0$ or b_n accordingly as n is even or odd.

Type (ii) : Only one wall, say the lower one, is ITBT-affected. This case corresponds to $\Phi = -i\infty$. Here $B_n(\lambda) = (1/2)b_n$, for integer values of n, and

$$b_n = \frac{-4n\pi^2\lambda^2}{\left[\lambda^2 + (n+1)^2\pi^2\right]\ \left[\lambda^2 + (n-1)^2\pi^2\right]}. \tag{36}$$

$$\lambda = (1-i)\ \left(\frac{\omega}{2}\right)^{\frac{1}{2}}, |b_n|^2 = \frac{16n^2\pi^4\omega^2}{\left[\omega^2 + (n+1)^4\pi^4\right]\ \left[\omega^2 + (n-1)^4\pi^4\right]}. \tag{37}$$

Following Venezian [1], we get the expression for R_{2c} in the form :

$$R_{2c} = \frac{-R_0\left[\pi^2+a^2\right]-La^2}{4[\pi^2+a^2]} \times \sum \left[R_0 a^2\left(n^2\pi^2+a^2\right) + La^4\right]\ |b_n|^2 C_n \quad , \tag{38}$$

where

$$C_n = \frac{L_1(\omega,n) + L_1^*(\omega,n)}{2\left|L_1(\omega,n)\right|^2}$$

$$= \left(\left(n^2\pi^2+a^2\right)^2\left(\left(n^2\pi^2+a^2\right)^2 - \frac{\omega^2}{\mathrm{Pr}}\right) - 2La^4 - 2a^2\left(n^2\pi^2+a^2\right)R_0\right)\Big/ d_n,$$

$$d_n = \left\{ \left(n^2\pi^2 + a^2\right)^2 \left(\left(n^2\pi^2 + a^2\right)^2 - \frac{\omega^2}{\mathrm{Pr}} \right) - 2La^4 - 2a^2 \left(n^2\pi^2 + a^2\right) R_0 \right\}^2 + \left\{ \omega \left(1 + \frac{1}{\mathrm{Pr}}\right) \left(n^2\pi^2 + a^2\right)^3 \right\}^2 . \tag{39}$$

In equation (38), the summation extends over even values of n for case (a), odd values of n for case (b), type (i), and for all integer values of n for case (b), type (ii). The infinite series (38) converges rapidly in all cases.

3 Results and Discussion

We make an analytical study of the effects of dielectric-related parameters and time-periodic boundary temperatures on the onset of convection in a Newtonian dielectric fluid. It was found in this case that R_{2c} is a crucial quantity which determines whether ITBT leads to sub-critical instability or not. The study of the behaviour of R_{2c} is of some interest in the limiting cases of very small and very large frequencies. We find that when $\omega >> 1$, R_{2c} tends to zero, so that the effects of ITBT and the dielectric parameters become small. For moderate values of ω, the dielectric parameter will affect R_{2c}. In the paper we consider two types of ITBT

1. Synchronous ITBT which means that the two ITBTs are in-phase. ($\Phi = 0$)
2. Asynchronous ITBT which means that the two ITBTs are out-of-phase. In this case we consider two sub-cases:
   i) There is phase difference between the two ITBTs ($\Phi = \pi$) and
   ii) Only one wall, say the lower one, is ITBT-affected. *($\Phi \to -i\infty$)*

Figure 2 shows the variation of R_{2c} with ω for different values of Pr (with L fixed) in the case of synchronous ITBT. It may be noticed that for moderate values of frequency, R_{2c} decreases with an increase in Pr. We can infer from this that the effect of increasing Pr is to destabilize the system. It is appropriate to note here that Pr does not affect the R_0-part of R. It affects only R_2. It is also observed that for low concentration of the suspended dielectric particles supercritical motion is possible and for high concentration only subcritical motion is possible. Thus, in the case of fluids with suspended particles subcritical motions are more probable than supercritical motions.

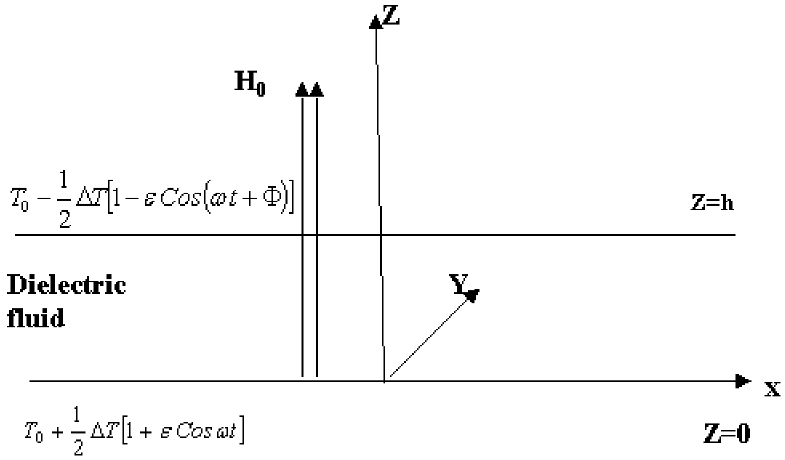

Figure 1: Physical configuration of the Rayleigh-Benard convection for a Dielectric fluid with imposed time-periodic boundary temperatures.

Figure 3 is the plot of R_{2c} versus ω for different values of the electric Rayleigh number L (the Prandtl number Pr being fixed) in respect of synchronous ITBT. The electric Rayleigh number L is the ratio of the electric to the gravitational forces. We see from the figure that for synchronous ITBT when L is greater than 2972 super critical motion occurs and R_{2c} increases with an increase in L at a given frequency ω. Hence, L has a stabilizing effect on the flow. It is also interesting to see from the figures that for given L $(L < 2972)$, R_{2c} first decreases with increase in ω, reaches a minimum and then increases with increase in $\omega$and for a given L $(L > 2972)$, R_{2c} first increases with increase in ω reaches a maximum and then decreases with increase in ω. This shows that for a weakly dielectric fluid, the flow is destabilized for small values of ω and stabilized for large ω. This is due to the fact that when the frequency of modulation is low, the effect of ITBT is felt throughout the fluid. For synchronous ITBT of the fluid, the temperature profiles consists of the steady line section plus a parabolic profile which oscillate in time. As the amplitude of the modulation increases the parabolic part of the profile becomes more and more significant. It is known that a parabolic profile is subject to finite amplitude instabilities so that convection occurs at lower Rayleigh numbers than those predicted by the linear theory. There is also a value of ω for which the stabilizing influence is minimum and this minimum decreases

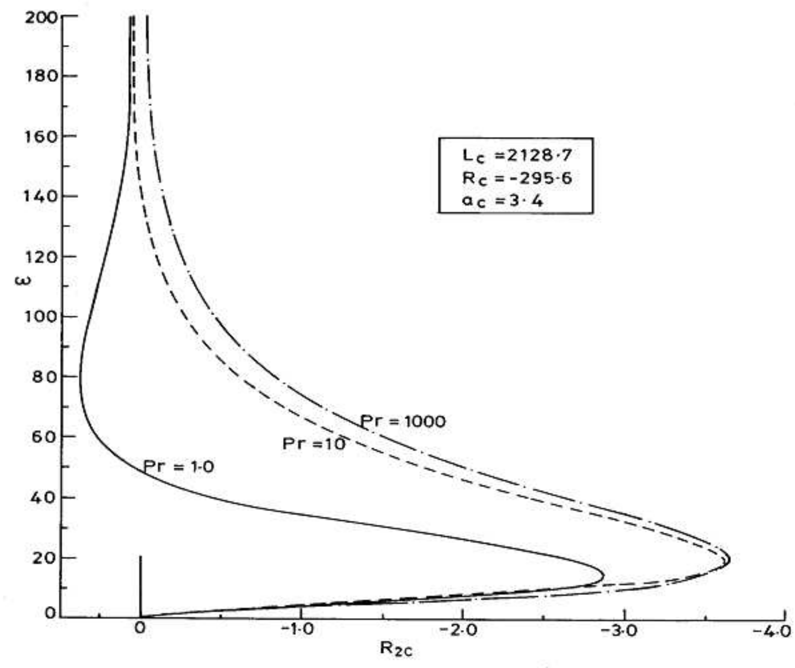

Figure 2: R_{2c} as a function of ω when the wall temperatures are modulated in phase for $Pr = 1$, $Pr = 10$ and $Pr = 100$.

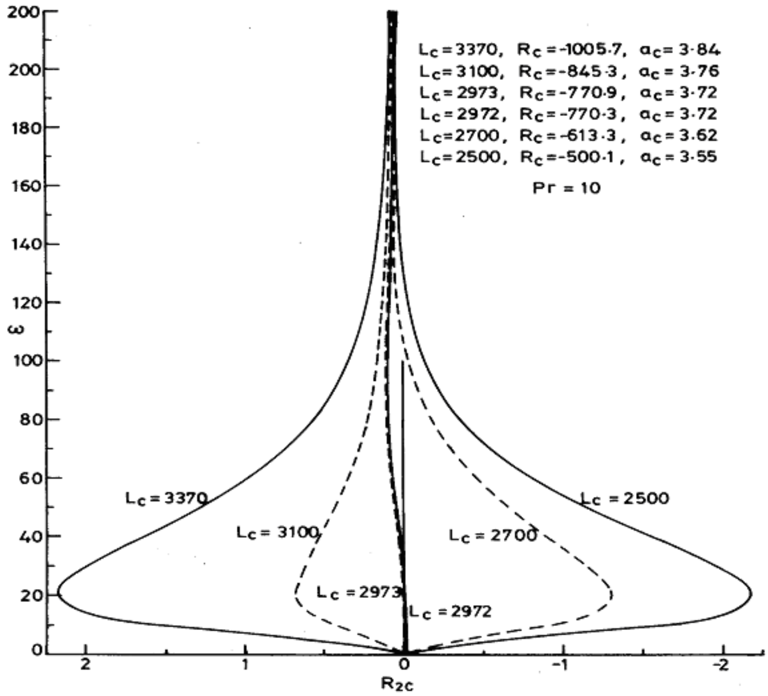

Figure 3: R_{2c} as a function of ω when the wall temperatures are modulated in phase for different values of the critical electric Rayleigh number.

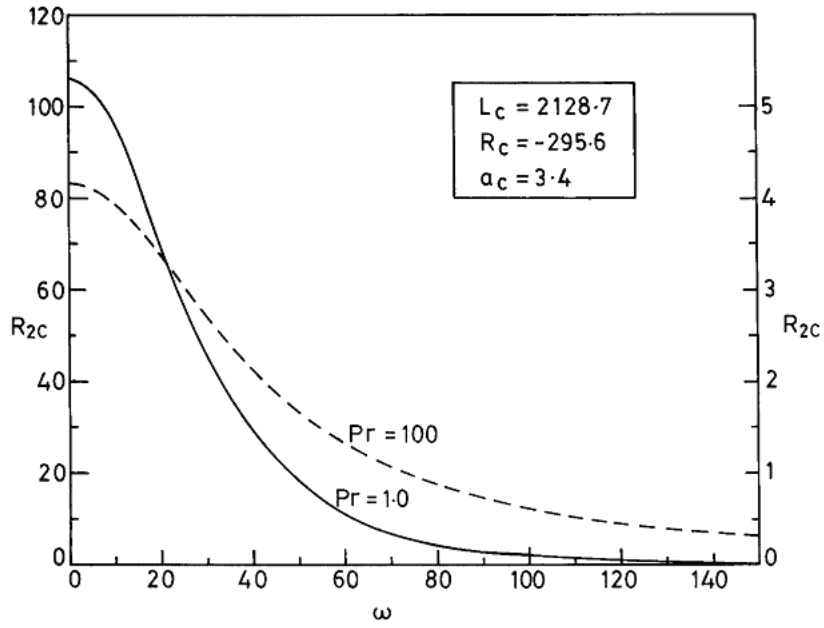

Figure 4: R_{2c} as a function of ω when the wall temperatures are modulated out of phase for $Pr = 1$ and $Pr = 100$.

with an increase in L.

We observe from Figure 4 that for asynchronous ITBT with a phase difference between the two ITBTs, even though R_{2c} decreases with an increase in L and Pr it does not become negative. Thus subcritical motions are ruled out in this case. The above results are due to the fact that in the case of asynchronous ITBT the temperature field has essentially a linear gradient varying in time, so that the instantaneous Rayleigh number is supercritical for half a cycle and subcritical during the other half cycle (see Venezian (1969)). We also observe that L and Pr have opposing influences in synchronous and asynchronous ITBT.

For asynchronous ITBT where only the lower wall is ITBT-affected we observe from Figure 5 that the effect of the various parameters on R_{2c} is qualitatively similar to the previous case of asynchronous ITBT with a phase difference between the two ITBTs. A point to be noted in this case is that for very high values of the Prandtl number Pr, subcritical motions are possible for low and moderate values

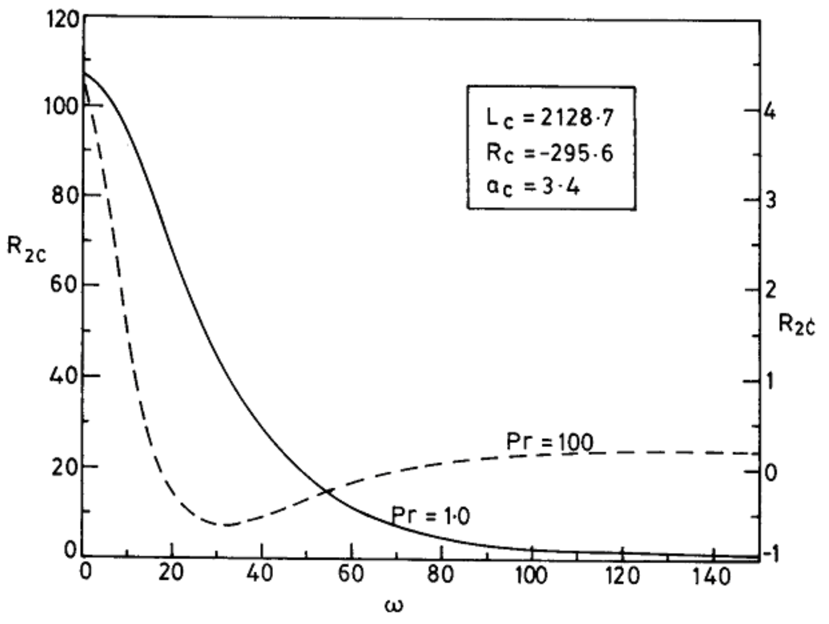

Figure 5: R_{2c} as a function of ω when the temperatures of the lower wall is modulated for $Pr = 1$ and $Pr = 100$.

of the frequency.

L	Pr	R_{0c}	R_{2c}	
			ω=10	ω=20
2128.7	10	-295.63	-2.667	-1.81
3000		-786.62	0.102	0.135

Table 1: Variation of R_{0c} and R_{2c} for different values of L in the case of synchronous ITBT.

We see from Table that R_c decreases more steeply than R_{2c} for the dielectric parameter.

4 Conclusion

The results of the study reaffirm the findings of Venezian (1969) for Newtonian fluids. The study indicates that ITBT can give rise to sub-critical motion. It is also observed that for large frequencies the effects of ITBT disappear. The problem throws light on an external means of controlling convection in dielectric liquids which is quite important from the application point of view.

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