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Unsteady MHD Flow Past a Vertical Oscillating Plate with Thermal Radiation and Variable Mass Diffusion

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Abstract: This investigation is undertaken to study the hydromagnetic flow of a viscous incompressible fluid past an oscillating vertical plate with radiation and variable mass diffusion. Governing equations are solved by Laplace transform technique for concentration, temperature and vertical velocity field and they are presented graphically for different values of physical parameters involved. It is observed that plate oscillation, variable mass diffusion and radiation affect the flow pattern significantly.

Keywords: Oscillating Plate, Radiation, Variable Mass Diffusion, MHD

2000 Mathematics Subject Classification: 76R10, 76W05, 80A20

1 Introduction

The study of radiative heat and mass transfer in convective flows is important from many industrial and technological points of view. Mass transfer is one of the most commonly encountered phenomena in chemical industry as well as in physical and biological sciences. When mass transfer takes place in a fluid at rest,

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the mass is transferred purely by molecular diffusion resulting from concentration gradients. For low concentration of the mass in the fluid and low mass transfer rates, the convective heat and mass transfer processes are similar in nature. A number of investigations have already been carried out with combined heat and mass transfer under the assumption of different physical situations. Radiation in free convection has also been studied by many authors because of its applications in many engineering and industrial processes. Examples include nuclear power plant, solar power technology, steel industry, fossil fuel combustion, space sciences applications, etc.

On the other hand, hydromagnetic flow is encountered in many applications such as in heat exchangers, in nuclear engineering, MHD accelerators etc. Due to some important industrial and engineering applications such as liquid metal cooling in nuclear reactors, magnetic control of molten iron flow in steel industry etc., magnetoconvection has been gaining considerable attention amongst researchers.

Exact solution of free convection flow past a vertical oscillating plate in free convective flow was first obtained by Soundalgekar [14] and the same problem with mass transfer effect was considered by Soundalgekar and Akolkar [15]. Das et. al. [2] studied the effects of mass transfer on free convection flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. The effects of mass transfer on the flow past an infinite vertical oscillating plate with constant heat flux was studied by Soundalgekar et. al. [16].

On the other hand, England and Emery [5] have considered the thermal radiation effects on the laminar free convection boundary layer of an absorbing gas bounded by a vertical stationary plate. Gupta and Gupta [6] studied the effect of radiation on the combined free and forced convection of an electrically conducting fluid flowing inside an open ended vertical channel in the presence of a uniform transverse magnetic field. Soundalgekar and Takhar [17] have studied the radiation effect on free convection flow past a vertical plate using Cogly-Vincentine-Gilles [1] equilibrium model. Das et. al. [3] considered the case of radiation effects on flow past an impulsively started vertical plate. Hosain and Takhar [9] studied the radiation effect on mixed convection along a vertical plate with uniform surface temperature while Mazumdar and Deka [11] considered the MHD flow past an impulsively started infinite vertical plate in presence of thermal radiation.

The effects of mass transfer on free convection flow past a vertical isothermal plate was first studied by Gebhart and Pera [8]. Oscillating plate with thermal radiation was studied by Muthucumaraswamy [13] with variable temperature and mass diffusion. MHD and radiation with variable mass diffusion was considered by Muthucumaraswamy and Janakiraman [12]. Deka and Neog [4] studied the combined effects of thermal radiation and chemical reaction on free convection flow past a vertical plate in porous medium. All of them considered the fact that free convection current caused by temperature differences is also caused by the differences in concentration or material constitution as suggested by Gebhart [7].

Although different authors studied mass transfer with or without radiation effects on the flow past oscillating vertical plate by considering different surface conditions but the study on the effects of magnetic field on free convection heat and mass transfer with thermal radiation and variable mass diffusion in flow through an oscillating plate has not been found in literature and hence the motivation to undertake this study. It is therefore proposed to study the effects of thermal radiation and variable mass diffusion on hydromagnetic flow past an oscillating vertical plate.

2 Mathematical Analysis

We consider an unsteady natural convection flow of a viscous incompressible electrically conducting fluid past an infinite vertical plate. To visualize the flow pattern a Cartesian co-ordinate system considered where x'-axis is taken along the infinite vertical plate, the y'-axis is normal to the plate and fluid fills the region $y' \geq 0$ [ref. Figure 1].

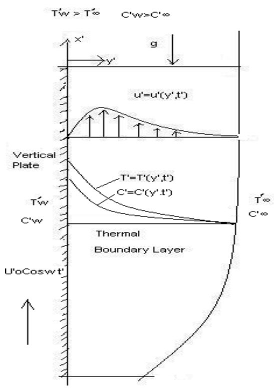

Figure 1: Flow Diagram

Initially, the fluid and the plate are kept at the same constant temperature T'_∞ and species concentration C'_∞. At time $t' > 0$, the plate is given an oscillatory motion in its own plane with a velocity $U_0 \cos\omega' t'$. At the same time the plate temperature is raised to T'_w and concentration is raised linearly with time and a magnetic field of uniform strength B_0 is applied normal to the plate. It is assumed that the magnetic Reynolds number is very small and the induced magnetic field is negligible in comparison to the transverse magnetic field. It is also assumed that the effect of viscous dissipation is negligible in the energy equation and the level of species concentration is very low so that the Soret and Dufour effects are negligible.

As the plate is infinite in extent so the derivatives of all the flow variables with respect to x' vanish and they can be assumed to be functions of y' and t' only. Thus the motion is one dimensional with only non-zero vertical velocity component u', varying with y' and t' only. Due to one dimensional nature, the equation of continuity is trivially satisfied.

Under the above assumptions and following Oberbeck-Boussinesq approximation, the unsteady flow field is governed by the following set of equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2 u'}{\rho} \tag{1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

along with the following initial and boundary conditions:

$$\begin{aligned}
&\text{For } t' \le 0: u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty, \quad \text{for all } y' \\
&\text{For } t' > 0,\ u' = U_0\cos\omega' t', \quad T' = T'_w, \quad C' = C'_\infty + (C'_w - C'_\infty)At', \ \text{at } y' = 0 \\
&\text{and} \qquad u' \to 0, \quad T' \to T'_\infty \quad C' \to C'_\infty \quad \text{as} \quad y' \to \infty, \qquad \text{where } A = \frac{U_0}{\nu}
\end{aligned} \tag{4}$$

We assume that the medium is optically thin with relatively low density. Thus following Cogly-Vincentine-Gilles equilibrium model we have

$$\frac{\partial q_r}{\partial y'} = 4(T' - T'_\infty)\int_0^\infty K_w \left(\frac{\partial e_b}{\partial T'}\right)_w d\lambda = 4I^*(T' - T'_\infty) \tag{5}$$

With the help of (5), equation (2) can be rewritten as follows:

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} - 4I^*(T' - T'_\infty) \tag{6}$$

Now to reduce the governing equations in dimensionless form we introduce the following quantities:

$$\begin{gathered} u = \frac{u'}{U_0},\ t = \frac{t'U_0^2}{\nu},\ y = \frac{y'U_0}{\nu},\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \\ \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ Pr = \frac{\rho\nu C_p}{k},\ Sc = \frac{\nu}{D},\ Gr = \frac{g\beta\nu(T'_w - T'_\infty)}{U_0^3}, \\ Gm = \frac{g\beta^*\nu(C'_w - C'_\infty)}{U_0^3},\ M = \frac{\sigma B_0^2\nu}{\rho U_0^2},\ F = \frac{4I^*\nu^2}{kU_0^2} \end{gathered} \tag{7}$$

Thus the equations (1), (6) and (3) reduce to:

$$\frac{\partial u}{\partial t} = Gr\theta + Gm\phi + \frac{\partial^2 u}{\partial y^2} - Mu \tag{8}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} - \frac{F}{Pr}u \tag{9}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} \tag{10}$$

And the initial and boundary conditions (4) then reduce to:

$$\begin{aligned} \text{For } t \le 0 : u = 0, \quad \theta = 0, \quad \phi = 0 \quad &\text{for all } y \\ \text{For } t > 0 : u = \cos\omega t, \quad \theta = 1, \quad \phi = t \quad &\text{at } y = 0 \\ u \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad &\text{as } y \to \infty \end{aligned} \tag{11}$$

Solutions of the equations (8), (9) and (10) subject to the initial and boundary conditions (11) are obtained by Laplace transform technique. Thus using Hetnarski's [10] algorithm, we obtain the solutions for u, θ and ϕ as follows:

$$\begin{aligned}
u(y,t) = {} & \frac{1}{4}\left[e^{i\omega t}\left\{e^{-y\sqrt{M'}}\ erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{M't}\right)+e^{y\sqrt{M'}}\ erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{M't}\right)\right\}+cc\right] \\
& +\frac{1}{2}\left(G_3-\frac{tG_2}{d}\right)\left[\left\{e^{-y\sqrt{M}}\ erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)+e^{y\sqrt{M}}\ erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\right\}\right] \\
& +\frac{yG_2}{4d\sqrt{M}}\left[\left\{e^{-y\sqrt{M}}\ erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)-e^{y\sqrt{M}}\ erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\right\}\right] \\
& -\frac{G_1}{2b}\left[e^{-bt}\left(\left\{e^{-y\sqrt{c}}\ erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{ct}\right)+e^{y\sqrt{c}}\ erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{ct}\right)\right\}\right.\right. \\
& \left.\left.-\left\{e^{-y\sqrt{fPr}}\ erfc\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}-\sqrt{ft}\right)+e^{y\sqrt{fPr}}\ erfc\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}+\sqrt{ft}\right)\right\}\right)\right] \\
& -\frac{G_1}{2b}\left\{e^{-y\sqrt{aPr}}\ erfc\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}-\sqrt{at}\right)+e^{y\sqrt{aPr}}\ erfc\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}+\sqrt{at}\right)\right\} \\
& +\frac{G_2}{d^2}\left[e^{dt}\left(\left\{e^{-y\sqrt{h}}\ erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{ht}\right)+e^{y\sqrt{h}}\ erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{ht}\right)\right\}\right.\right. \\
& \left.-\left\{e^{-y\sqrt{dSc}}\ erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{dt}\right)+e^{y\sqrt{dSc}}\ erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{dt}\right)\right\}\right) \\
& \left.+\left(1+dt+\frac{y^2dSc}{2}\right)\ erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}\right)-\frac{yd\sqrt{tSc}}{\sqrt{\pi}}e^{\frac{-y^2Sc}{4t}}\right]
\end{aligned} \tag{12}$$

$$\theta(y,t)=\frac{1}{2}\left[e^{-y\sqrt{aPr}}\ erfc\Big(\frac{y\sqrt{Pr}}{2\sqrt{t}}-\sqrt{at}\Big)+e^{y\sqrt{aPr}}\ erfc\Big(\frac{y\sqrt{Pr}}{2\sqrt{t}}+\sqrt{at}\Big)\right] \tag{13}$$

$$\phi(y,t)=\Big(t+\frac{y^2Sc}{2}\Big)erfc\Big(\frac{y\sqrt{Sc}}{2\sqrt{t}}\Big)-\frac{y\sqrt{tSc}}{\sqrt{\pi}}e^{\frac{-y^2Sc}{4t}} \tag{14}$$

The symbols used here are as follows:

$$\begin{gathered}
M'=M+i\omega,\quad a=\frac{F}{Pr},\quad b=\frac{F-M}{Pr-1},\quad c=M-b, \\
d=\frac{M}{Sc-1},\quad f=a-b,\quad h=M+d,\quad G_1=\frac{Gr}{Pr-1}, \\
G_2=\frac{Gm}{Sc-1},\quad G_3=\frac{G_1}{b}-\frac{G_2}{d^2},\quad cc=\text{complex conjugate}
\end{gathered} \tag{15}$$

3 Results and Discussion

In order to know the influence of different physical parameters viz., radiation parameter, Schmidt number, thermal Grashof number, mass Grashof number, Hartmann number, Prandtl number, phase angle and time on the physical flow field, computations are carrier out for vertical velocity, temperature and concentration and they are presented in figures below.

In Figure 2 concentration profile is presented for different values of Sc(0.6, 0.78, 1.5, 2.5) at time $t = 0.2$. It is observed that increase of Schmidt number leads to the decrease in concentration of the species.

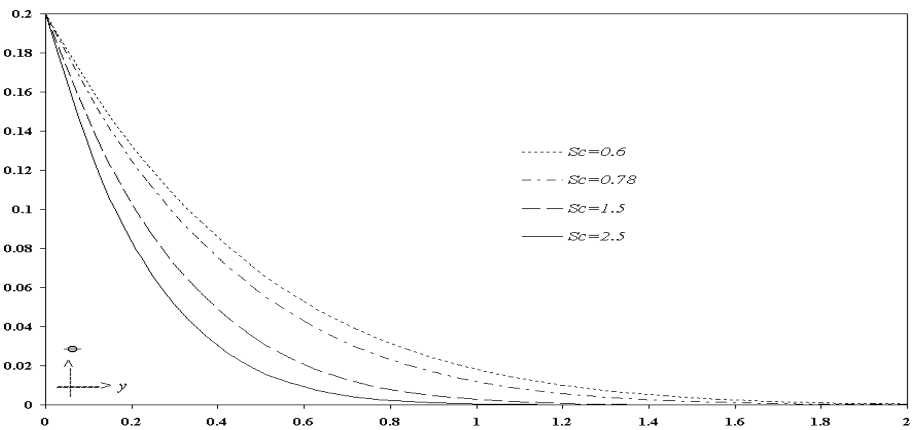

Figure 2: Concentration profiles showing the effect of Sc at t=0.2.

Figure 3 represents the concentration profile for different values of time t. Since concentration is considered as time dependent, therefore this figure clearly reflects the situation that is concentration increases with increasing time.

Effects of radiation parameter F on the temperature field is presented in Figure 4 for Prandtl number 7 and 0.71. It is clear from the figure that temperature decreases with the increase of radiation parameter. Also change of temperature for $Pr = 7$ is more rapid than $Pr = 0.71$.

Velocity profiles for different values of parameters are shown in Figures 5–8. Influence of $Gr(5, 10)$, $Gm(2, 5)$ and $M(0.5, 1)$ are shown in Figure 5 for some fixed values of $Sc(2.5)$, $F(2.5)$, $Pr(7)$, $t = 0.2$ and $\omega t(\pi/2)$. It is clear from the figure that velocity increases with the increase of Gr and Gm but decreases with the increases of M. In Figure 6 effects of $M(1, 3)$ and $F(0.5, 2.5)$ is presented for

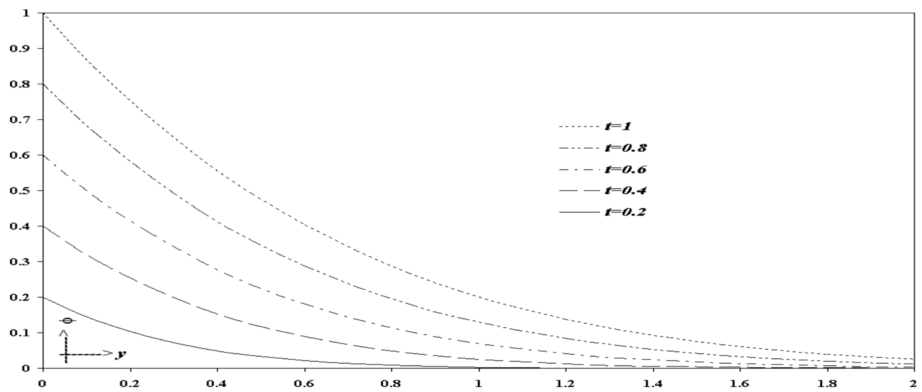

Figure 3: Concentration profiles for different values of t at Sc=1.5.

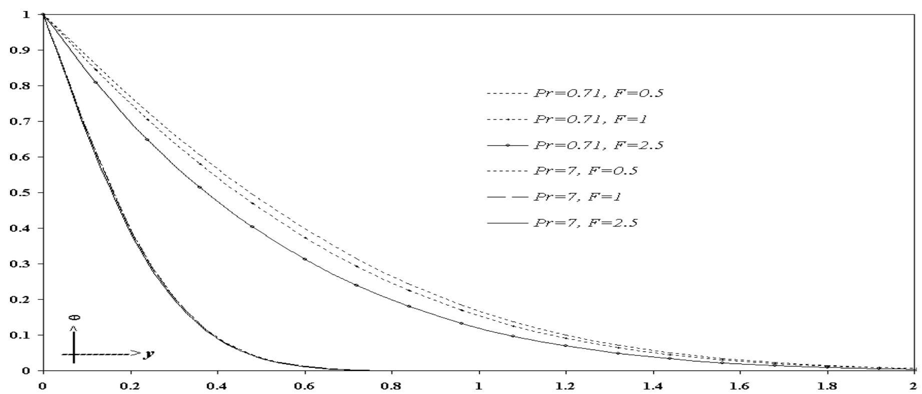

Figure 4: Temperature profiles for different values of F for Pr=7 and 0.71 at t=0.2.

some fixed values of $t(0.2)$, $Gr(10)$, $Gm(5)$, $Pr(7)$, $Sc(1.5)$ and $\omega t(\pi/2)$. It is clear from the figure that velocity decreases when M and F increase. Effects of $Sc(1.5, 2.5, 5)$ and $M(1, 2.5)$ for fixed values of $t(0.2), Gr(10), Gm(5), Pr(7), F(1)$ and $\omega t(\pi/2)$ is presented in Figure 7 from which it is clear that increase of Schmidt number Sc and the Hartmann number M lead to the decrease of velocity. Velocity profile for different values of phase angle ωt are presented in Figure 8 for some fixed values $Gr(10), Gm(5), Pr(7), M(0.5), Sc(1.5), F(1)$ and $t(0.2)$. Increase of phase angle ωt leads to decrease of velocity.

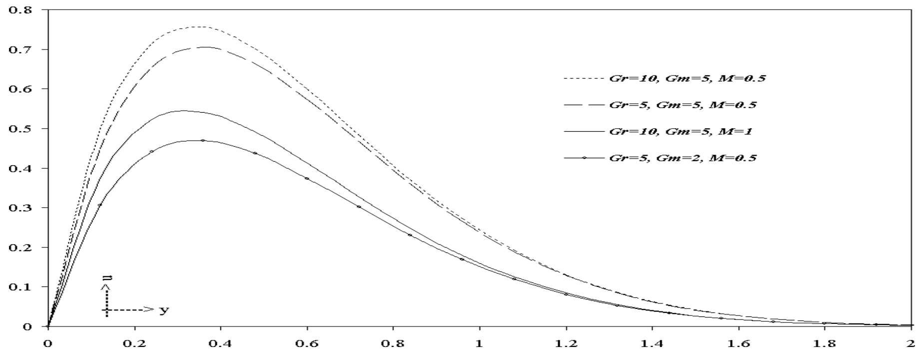

Figure 5: Velocity profiles for different values of *Gr, Gm* and *M* at *t*=0.2.

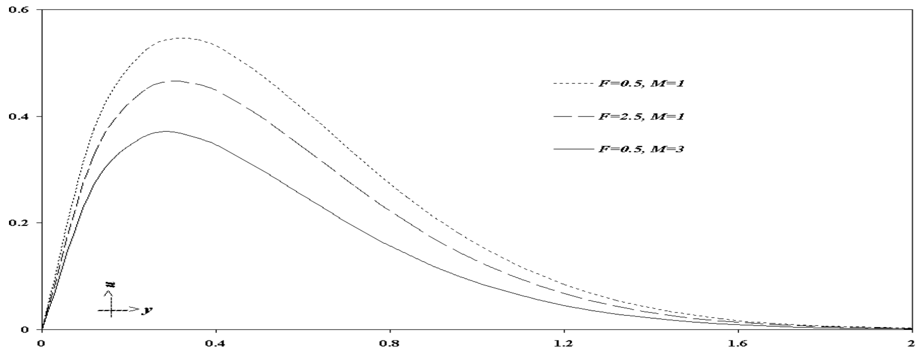

Figure 6: Velocity profiles for different values of *M* and *F* at *t*=0.2.

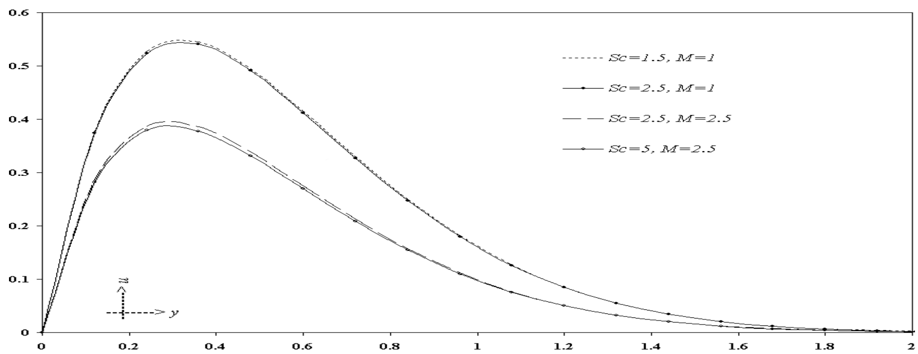

Figure 7: Velocity profiles for different values of *Sc* and *M* at *t*=0.2.

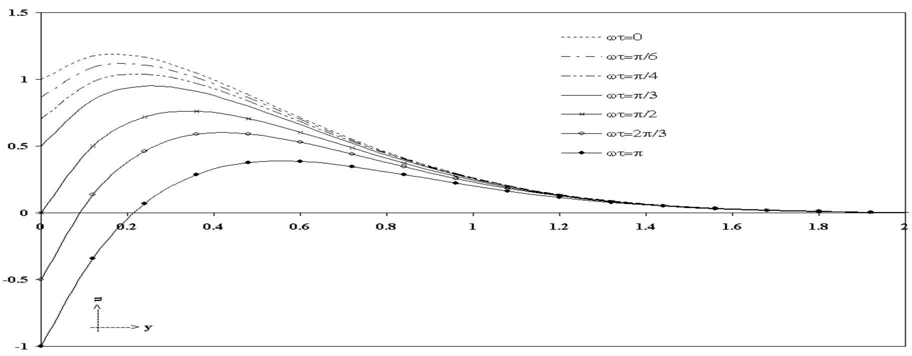

Figure 8: Velocity profiles for different values of phase angle ωt.

Skin-friction: To study the effects of mass transfer on the thermal boundary layer we have also evaluated the skin-friction, which is given in non-dimensional form by

$$\tau = -\left(\frac{du}{dy}\right)_{y=0}$$

Thus from (12), we obtain the expression for τ as follows-

$$\begin{aligned}\tau = &\frac{1}{2}[e^{i\omega t}\sqrt{M'}\ erf(\sqrt{M't}) + cc] + G_3\sqrt{M}\ erf(\sqrt{Mt}) + G_4\frac{e^{-Mt}}{\sqrt{\pi t}} \\ &-\frac{G_1}{b}[e^{-bt}\{\sqrt{c}\ erf(\sqrt{ct}) - \sqrt{fPr}\ erf(\sqrt{ft})\} + \sqrt{aPr}\ erf(\sqrt{at})] \\ &-\frac{G_2}{d}[(t\sqrt{M} + \frac{1}{2\sqrt{M}})\ erf(\sqrt{Mt}) + \sqrt{\frac{t}{\pi}}(e^{-Mt} - 2\sqrt{Sc})] \\ &+\frac{G_2}{d^2}[e^{-dt}\{\sqrt{h}\ erf(\sqrt{ht}) - \sqrt{dSc}\ erf(\sqrt{dt})\}] \qquad (16)\end{aligned}$$

where $G_4 = 1 + G_3 - \frac{G_1}{b} + \frac{G_2}{d^2}$. Computed values of τ are presented in the Table below. It is observed that the skin-friction decreases with the increase of Gr, Gm and t and increases with the increase of Sc, M and F.

Pr	t	Gr	Gm	Sc	M	F	τ
7	0.2	10	5	2.5	0.5	2	-3.2580060
7	0.2	5	5	2.5	0.5	2	-2.5814950
7	0.2	5	5	2.5	1.0	2	-2.5237500
7	0.2	5	5	2.5	0.5	3	-2.5769260
7	0.2	5	5	2.5	1.5	3	-2.4644590
7	0.2	5	5	2.0	0.5	2	-2.5903690
7	0.4	5	5	2.5	0.5	2	-2.5183370
7	0.2	5	10	2.5	0.5	2	-2.7108290

4 Conclusion

A theoretical analysis is performed to study the influence of thermal radiation on hydromagnetic flow past a vertical oscillating plate with variable mass diffusion. Exact solutions of equations are obtained by Laplace transform technique. Some of the important conclusions of the study are as follows-

- Concentration decreases as Sc increases.
- Velocity increases with the increase of Gr and Gm and decreases with the increase of M and F increase.
- Increase of Sc, F and t lead to the decrease of velocity.
- Skin-friction increases with increasing M and F.

NOMENCLATURE

B_0	Magnetic field strength alongy'-axis
C_p	Specific heat at constant pressure
C'	Species concentration in the fluid
C'_w	Concentration on the plate
C'_∞	Concentration of the main stream fluid
D	Mass diffusion coefficient
e_b	Plank function
F	Dimensionless thermal radiation parameter
g	Acceleration due to gravity
Gr	Thermal Grashof number
Gm	Mass Grashof number

k	Thermal conductivity
K_w	Absorption coefficient
M	Hartmann number
Pr	Prandtl number
q_r	Radiative heat flux
Sc	Schmidth number
T'	Temperature of fluid near the plate
T'_w	Temperature of the plate
T'_∞	Temperature of the mainstream fluid
t'	Time
t	Dimensionless Time
u'	Velocity components in the x-direction
u	Dimensionless velocity
U_o	Velocity of the plate
x'	Co-ordinate along the plate
y'	Co-ordinate normal to the plate
x	Dimensionless co-ordinate along the plate
y	Dimensionless co-ord. normal to the plate
erf	error function
$erfc$	complementary error function

Greek Letters

β	Co-efficient of thermal expansion
β^*	Co-efficient of expansion with concentration
θ	Dimensionless temperature
ϕ	Dimensionless Concentration
μ	Co-efficient of viscosity
ν	Kinematic viscosity
ρ	Fluid density
σ	Electrical conductivity
τ	Dimensionless skin-friction
ωt	Dimensionless Phase angle
$\omega' t'$	Phase angle

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