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Neighborhood Connected 2-Domination Number and Connectivity of Graphs

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Abstract: A subset S of V is called a dominating set in G if every vertex in $V-S$ is adjacent to at least one vertex in S. A set $S \subseteq V$ is called the neighborhood connected 2-dominating set (nc2d-set) of a graph G if every vertex in $V-S$ is adjacent to at least two vertices in S and the induced subgraph $\langle N(S) \rangle$ is connected. The minimum cardinality of a nc2d-set of G is called the neighborhood connected 2-domination number of G and is denoted by $\gamma_{2nc}(G)$. The connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected or trivial graph. In this paper we find an upper bound for the sum of the neighborhood connected 2-domination number and connectivity of a graph and characterize the corresponding extremal graphs.

Keywords: Neighborhood connected 2-domination number, Connectivity

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1 Introduction

By a graph $G = (V, E)$ we mean a finite, undirected and connected graph with neither loops nor multiple edges. The order and size of G are denoted by n and m respectively. The degree of any vertex u in G is the number of edges incident with u and is denoted by $deg(u)$. The minimum and maximum degree of a graph

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G is denoted by $\delta(G)$ and $\Delta(G)$, respectively. For graph theoretic terminology we refer to Chartrand and Lesniak [1] and Haynes et.al [2, 3].

Let $v \in V$. The open neighborhood and closed neighborhood of v are denoted by $N(v)$ and $N[v] = N(v) \cup \{v\}$ respectively. If $S \subseteq V$ then $N(S) = \bigcup_{v \in S} N(v)$ for all $v \in S$ and $N[S] = N(S) \cup S$. If $S \subseteq V$ and $u \in S$ then the private neighbor set of u with respect to S is defined by $\text{pn}[u, S] = \{v : N[v] \cap S = \{u\}\}$. $H(m_1, m_2, \cdots, m_n)$ denotes the graph obtained from the graph H by attaching m_i edges to the vertex $v_i \in V(H)$, $1 \leq i \leq n$. $H(P_{m_1}, P_{m_2}, \cdots, P_{m_n})$ is the graph obtained form the graph H by attaching the end vertex of P_{m_i} to the vertex v_i in H, $1 \leq i \leq n$.

A subset S of V is called a dominating set of G if every vertex in $V - S$ is adjacent to at least one vertex in S. The minimum cardinality taken over all dominating sets in G is called the domination number of G and is denoted by $\gamma(G)$. The same authors introduced in [5] the concept of neighborhood connected 2-domination in graphs. A set $S \subseteq V$ is called a neighborhood connected 2-dominating set (nc2d-set) of a graph G if every vertex in $V - S$ is adjacent to at least two vertices in S and the induced subgraph $\langle N(S) \rangle$ is connected. The minimum cardinality of a nc2d-set of G is called the neighborhood connected 2-domination number of G and is denoted by $\gamma_{2nc}(G)$. The connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected or trivial graph.

Several authors have studied the problem of obtaining an upper bound for the sum of a domination parameter and a graph theoretic parameter and characterized the corresponding extremal graphs. J. Paulraj Joseph and S. Arumugam [4] proved that $\gamma(G) + \kappa(G) \leq n$ and characterized the corresponding extremal graphs. In this paper, we obtain a sharp upper bound for the sum of the neighborhood connected 2-domination number and connectivity of a graph and characterize the corresponding extremal graphs. We use the following theorems.

Theorem 1.1. [5] *For any graph G, $\gamma_{2nc}(G) \leq n$ and equality holds if and only if G is isomorphic to K_2.*

Theorem 1.2. *For a graph G, $\kappa(G) \leq \delta(G)$.*

2 Main Results

Theorem 2.1. *For any graph* G, $\gamma_{2nc}(G) + \kappa(G) \leq 2n - 1$ *and equality holds if and only if* G *is isomorphic to* K_2.

Proof. $\gamma_{2nc}(G)+\kappa(G) \leq n+\delta \leq n+n-1 = 2n-1$. Let $\gamma_{2nc}(G)+\kappa(G) = 2n-1$. Then $\gamma_{2nc}(G) = n$ and $\kappa(G) = n - 1$ which gives G is isomorphic to K_2. The converse is obvious. □

Theorem 2.2. *For any graph* G, $\gamma_{2nc}(G) + \kappa(G) = 2n - 2$ *if and only if* G *is isomorphic to* K_3.

Proof. Let $\gamma_{2nc}(G) + \kappa(G) = 2n - 2$. Then there are two cases to consider (i) $\gamma_{2nc}(G) = n$ and $\kappa(G) = n - 2$ (ii) $\gamma_{2nc}(G) = n - 1$ and $\kappa(G) = n - 1$. Condition (i) is impossible. Hence condition (ii) holds. Since $\kappa(G) = n - 1$ then G is a complete graph. This gives $\gamma_{2nc}(G) = 2$. Then $n = 3$ and hence G is isomorphic to K_3. The converse is obvious. □

Theorem 2.3. *For any graph* G, $\gamma_{2nc}(G) + \kappa(G) = 2n - 3$ *if and only if* G *is isomorphic to* C_4 *or* $K_{1,2}$ *or* K_4.

Proof. Let $\gamma_{2nc}(G) + \kappa(G) = 2n - 3$. Then there are three cases to consider (i) $\gamma_{2nc}(G) = n$ and $\kappa(G) = n - 3$ (ii) $\gamma_{2nc}(G) = n - 1$ and $\kappa(G) = n - 2$ (iii) $\gamma_{2nc}(G) = n - 2$ and $\kappa(G) = n - 1$.

Case 1. $\gamma_{2nc}(G) = n$ and $\kappa(G) = n - 3$.

There is no graph that satisfies this condition.

Case 2. $\gamma_{2nc}(G) = n - 1$ and $\kappa(G) = n - 2$.

Then $n - 2 \leq \delta(G)$. If $\delta = n - 1$ then G is a complete graph which gives a contradiction. Hence $\delta(G) = n-2$. Then G is isomorphic to $K_n - Y$ where Y is a matching in G. Then $\gamma_{2nc}(G) \leq 3$. If $\gamma_{2nc}(G) = 3$ then $n = 4$ and hence G is isomorphic to C_4. If $\gamma_{2nc}(G) = 2$ then $n = 3$ and hence G is isomorphic to $K_{1,2}$.

Case 3. $\gamma_{2nc}(G) = n - 2$ and $\kappa(G) = n - 1$.

Then G is a complete graph on n vertices. Since $\gamma_{2nc} = 2$ we have $n = 4$. Hence G is isomorphic to K_4. The converse is obvious. □

Theorem 2.4. *For any graph* G, $\gamma_{2nc}(G) + \kappa(G) = 2n - 4$ *if and only if* G *is isomorphic to* P_4 *or* K_5 *or* $K_4 - e$ *or* $K_{1,3}$ *or* $K_3(1, 0, 0)$.

Proof. Let $\gamma_{2nc}(G) + \kappa(G) = 2n - 4$. Then there are four cases to consider (i) $\gamma_{2nc}(G) = n$ and $\kappa(G) = n - 4$ (ii) $\gamma_{2nc}(G) = n - 1$ and $\kappa(G) = n - 3$ (iii) $\gamma_{2nc}(G) = n - 2$ and $\kappa(G) = n - 2$ (iv) $\gamma_{2nc}(G) = n - 3$ and $\kappa(G) = n - 1$.

Case 1. $\gamma_{2nc}(G) = n$ and $\kappa(G) = n - 4$.

There is no graph that satisfies this condition.

Case 2. $\gamma_{2nc}(G) = n - 1$ and $\kappa(G) = n - 3$.

Then $n - 3 \leq \delta$. If $\delta = n - 1$ then G is a complete graph which is a contradiction. If $\delta = n-2$ then G is isomorphic to $K_n - Y$ where Y is a matching in K_n. Then $\gamma_{2nc}(G) = 2$ or 3. If $\gamma_{2nc}(G) = 3$ then $n = 4$. Hence G is either $K_4 - e$ or C_4. For these two graphs $\kappa(G) = 2 \neq n - 3$ which is a contradiction. If $\gamma_{2nc}(G) = 2$ then $n = 3$ which gives a contradiction. Hence $\delta = n - 3$. Let X be the vertex cut of G with $|X| = n - 3$ and let $V - X = \{x_1, x_2, x_3\}$, $X = \{v_1, v_2, v_3, \ldots, v_{n-3}\}$.

Subcase 2.1. $\langle V - X \rangle = \overline{K_3}$.

Then every vertex of $V - X$ is adjacent to all the vertices in X. Suppose $E(\langle X \rangle) = \phi$ then $|X| \leq 3$ and hence G is isomorphic to $K_{3,3}$ or $K_{2,3}$ or $K_{1,3}$. But $\gamma_{2nc}(K_{3,3}) = 4 \neq n - 1$ and $\gamma_{2nc}(K_{2,3}) = 3 \neq n - 1$. Hence G is isomorphic to $K_{1,3}$. Suppose $E(\langle X \rangle) \neq \phi$. If any $v_1 \in X$ is adjacent to all the vertices in X and hence $\gamma_{2nc}(G) = 2$ then $n = 3$ which is impossible. Hence every vertex in X is not adjacent to at least one vertex in X. Hence $\gamma_{2nc}(G) = 3$. Then $n = 4$ which is also impossible.

Subcase 2.2. $\langle V - X \rangle = K_1 \cup K_2$.

Let $x_1 x_2 \in E(G)$. Then x_3 is adjacent to all the vertices in X and x_1, x_2 are not adjacent to at most one vertex in X. If $\deg x_1$ or $\deg x_2$ is $n - 2$ then $\{x_1, x_2, x_3\}$ is a nc2d-set of G and hence $\gamma_{2nc} \leq 3$. Then $n \leq 4$ which gives $n = 4$. Therefore G is isomorphic to P_4 or $K_3(1, 0, 0)$.

Suppose $\deg x_1 = \deg x_2 = n - 3$. If $N(x_1) = N(x_2)$ then there is a vertex $v_1 \in X$ such that v_1 is not adjacent to both x_1 and x_2. Then v_1 is adjacent to all the vertices in X. If $|X| \geq 4$ then $\{v_2, v_3\}$ is a nc2d-set of G and hence $n \leq 3$ which is a contradiction. If $|X| = 3$ then $\{v_1, v_2, v_3\}$ is a nc2d-set of G and hence $n = 4$ which is a contradiction. If $N(x_1) \neq N(x_2)$ then there are at least 2 vertices v_1 and v_2 such that v_1 is not adjacent to x_1 but adjacent to x_2 and v_2 is not adjacent to x_2 but adjacent to x_1. Then $\{x_1, x_2, x_3\}$ is an nc2d-set of G and hence $n \leq 4$ which gives a contradiction.

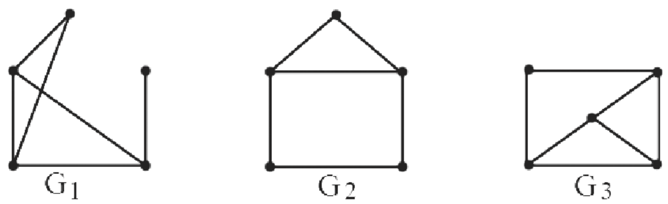
Case 3. $\gamma_{2nc}(G) = n - 2$ and $\kappa(G) = n - 2$.

Then $n - 2 \leq \delta(G)$. If $\delta = n - 1$ then G is a complete graph which gives a contradiction. Hence $\delta(G) = n-2$. Then G is isomorphic to $K_n - Y$ where Y is a matching in K_n. Then $\gamma_{2nc}(G) \leq 3$. If $\gamma_{2nc}(G) = 3$ then $n = 5$. But $\gamma_{2nc}(K_5 - Y) = 2 \neq n-2$ which is a contradiction. If $\gamma_{2nc}(G) = 2$ then $n = 4$. Hence G is isomorphic to $K_4 - e$.

Case 4. $\gamma_{2nc}(G) = n-3$ and $\kappa(G) = n-1$.

Then G is a complete graph on n vertices. Since $\gamma_{2nc}(G) = n-3$ we have $n = 5$. Hence G is isomorphic to K_5. The converse is obvious. □

Theorem 2.5. *For any connected graph G, $\gamma_{2nc}(G) + \kappa(G) = 2n-5$ if and only if G is isomorphic to any one of the following graphs (i) K_6 (ii) C_5 (iii) $K_{1,4}$ (iv) P_5 (v) $K_5 - Y$ where Y is any matching in K_5 (vi) $C_4(1,0,0,0)$ (vii) $K_2(1,2)$ (viii) $K_3(2,0,0)$ (ix) The graph G_i, $1 \leq i \leq 3$ given in the following figure.*



Proof. Let $\gamma_{2nc}(G) + \kappa(G) = 2n-5$. Then there are five cases to consider (i) $\gamma_{2nc}(G) = n$ and $\kappa(G) = n-5$ (ii) $\gamma_{2nc}(G) = n-1$ and $\kappa(G) = n-4$ (iii) $\gamma_{2nc}(G) = n-2$ and $\kappa(G) = n-3$ (iv) $\gamma_{2nc}(G) = n-3$ and $\kappa(G) = n-2$ (v) $\gamma_{2nc}(G) = n-4$ and $\kappa(G) = n-1$.

Case 1. $\gamma_{2nc}(G) = n$ and $\kappa(G) = n-5$.

There is no graph that satisfies this condition.

Case 2. $\gamma_{2nc}(G) = n-1$ and $\kappa(G) = n-4$.

Then $n-4 \leq \delta(G)$. If $\delta(G) = n-1$ then G is a complete graph which is a contradiction. If $\delta(G) = n-2$ then G is isomorphic to $K_n - Y$ where Y is a matching in K_n. Then $\gamma_{2nc}(G) \leq 3$. Then $n \leq 4$ which is a contradiction to $\kappa(G) = n-4$. Suppose $\delta(G) = n-3$. Let X be the vertex cut of G with $|X| = n-4$ and let $X = \{v_1, v_2, \ldots, v_{n-4}\}$, $V - X = \{x_1, x_2, x_3, x_4\}$. If $\langle V - X \rangle$ contains an isolated vertex then $\delta(G) \leq n-4$ which is a contradiction. Hence $\langle V - X \rangle$ is isomorphic to $K_2 \cup K_2$. Also every vertex of $V - X$ is adjacent to all the vertices of X. Then $\gamma_{2nc}(G) = 3$. Hence $n = 4$ which is a contradiction. Thus $\delta(G) = n-4$.

Subcase 2.1. $\langle V - X \rangle = \overline{K_4}$.

Then every vertex of $V-X$ is adjacent to all the vertices in X. Suppose $E(\langle X\rangle)=\phi$. Then $|X|\leq 4$ and hence G is isomorphic to $K_{s,4}$ where $s=1,2,3,4$. If $s=2,3$ or 4 then $\gamma_{2nc}(G)+\kappa(G)\neq 2n-5$. Hence G is isomorphic to $K_{1,4}$. Suppose $E(\langle X\rangle)\neq\phi$. If any one of the vertices in X say v_1 is adjacent to all the vertices in X we have that $\gamma_{2nc}(G)\leq 3$ which gives $n\leq 4$ which is a contradiction. Hence every vertex in X is not adjacent to at least one vertex in X. Hence $\gamma_{2nc}(G)\leq 4$. Then $n\leq 5$. Since $n\leq 4$ is impossible we have $n=5$ and hence G is isomorphic to $K_{1,4}$.

Subcase 2.2. $\langle V-X\rangle=P_3\cup K_1$.

Let x_1 be the isolated vertex in $\langle V-X\rangle$ and (x_2, x_3, x_4) be a path. Then x_1 is adjacent to all the vertices in X and x_2, x_4 are not adjacent to at most one vertex in X and hence $\{x_1, x_2, x_4, v_1\}$, $v_1\in X-N(x_2)$ is nc2d-set of G and hence $\gamma_{2nc}\leq 4$. Thus $n=5$. Then G is isomorphic to P_5 or $C_4(1,0,0,0)$ or $K_3(1,1,0)$ or $K_4-e(1,0,0,0)$. If G is either $K_3(1,1,0)$ or $K_4-e(1,0,0,0)$ then $\gamma_{2nc}(G)+\kappa(G)\neq 2n-5$. Hence G is P_5 or $C_4(1,0,0,0)$.

Subcase 2.3. $\langle V-X\rangle=K_3\cup K_1$.

Let x_1 be the isolated vertex in $\langle V-X\rangle$ and $\langle\{x_2, x_3, x_4\}\rangle$ be a complete graph. Then x_1 is adjacent to all the vertices in X and x_2, x_3, x_4 are not adjacent to at most two vertices in X and hence $\{x_1, x_2, x_3, v_1, v_2\}$ where $v_1, v_2\in X-N(x_2\cup x_3)$ is a nc2d-set of G and hence $n=5$ or 6. Suppose $n=5$. Then G is isomorphic to $K_4-e(1,0,0,0)$ or $K_4(1,0,0,0)$ or $K_3(P_3, P_1, P_1)$.

For these graphs $\gamma_{2nc}(G)+\kappa(G)\neq 2n-5$. Suppose $n=6$. Then $\{x_1, x_2, x_3, v_1\}$ or $\{x_1, x_2, x_3, v_2\}$ or $\{x_2, x_3, v_1, v_2\}$ is a nc2d-set of G which is a contradiction to $\gamma_{2nc}=n-1$.

Subcase 2.4. $\langle V-X\rangle=K_2\cup\overline{K_2}$.

Let $x_1x_2\in E(G)$ and $x_3x_4\in E(\overline{G})$. Then each x_i, $i=1$ or 2 is non adjacent to at most one vertex in X and each x_j, $j=3$ or 4 is adjacent to all the vertices in X. Then $\{x_1, x_3, x_4, v_1\}$ where $v_1\in N(x_2)\cap X$ is a nc2d-set of G and hence $n=5$. Then G is isomorphic to $K_2(2,1)$ or $K_3(2,0)$.

Subcase 2.5. $\langle V-X\rangle=K_2\cup K_2$.

Let $x_1x_2, x_3x_4\in E(G)$. Since $\delta(G)=n-4$ each x_i is non adjacent to at most one vertex in X. Then at most one vertex say $v_1\in X$ such that $|N(v_1)\cap(V-X)|=1$. If all $v_i\in X$ such that $|N(v_i)\cap(V-X)|\geq 2$ then $\{x_1, x_2, x_3, x_4\}$ is a nc2d-set of G and hence $n=5$. For this graph $\gamma_{2nc}(G)+\kappa(G)\neq 2n-5$. If $|N(v_1)\cap(V-X)|=1$ and $|N(v_i)\cap(V-X)|\geq 2$ for $i\neq 2$ then $\{x_1, x_2, x_3, x_4, v_1\}$ is a nc2d-set of G and hence $n=6$. For this graph

$\gamma_{2nc}(G) + \kappa(G) \neq 2n - 5$.

Case 3. $\gamma_{2nc}(G) = n - 2$ and $\kappa(G) = n - 3$.

Then $n - 3 \leq \delta$. If $\delta = n - 1$ then G is a complete graph which is a contradiction. If $\delta = n - 2$ then G is isomorphic to $K_n - Y$ where Y is any matching in K_n. Then $\gamma_{2nc}(G) = 2$ or 3. If $\gamma_{2nc}(G) = 3$ then $n = 5$ which gives a contradiction. If $\gamma_{2nc} = 2$ then $n = 4$. Hence G is either $K_4 - e$ or C_4. For these two graphs $\kappa(G) = 2 \neq n - 3$ which is a contradiction. Hence $\delta = n - 3$.

Let X be the vertex cut of G with $|X| = n - 3$ and let $V - X = \{x_1, x_2, x_3\}$, $X = \{v_1, v_2, \ldots, v_{n-3}\}$.

Subcase 3.1. $\langle V - X \rangle = \overline{K_3}$.

Then every vertex of $V - X$ is adjacent to all the vertices in X. Suppose $E(\langle X \rangle) = \phi$. Then $|X| \leq 3$. If $|X| = 1$ or 2 then $\gamma_{2nc}(G) + \kappa(G) \neq 2n - 5$ and hence G is isomorphic to $K_{3,3}$. If $E(\langle X \rangle) \neq \phi$. If any $v_1 \in X$ is adjacent to all the vertices in X then $\gamma_{2nc}(G) \leq 3$. Thus $n \leq 5$. If $n = 4$ then G is a star which is a contradiction to $E(\langle X \rangle) \neq \phi$. Hence $n = 5$. For this graph $\gamma_{2nc} = 2$ which is a contradiction. Hence there are no vertices of X of degree $n - 1$. Then $\gamma_{2nc}(G) \leq 4$ and hence $n = 6$. Hence G is isomorphic to the graph $K_{3,3} + e$. For this graph $\gamma_{2nc} = 3$ which is a contradiction.

Subcase 3.2. $\langle V - X \rangle = K_1 \cup K_2$.

Let $x_1 x_2 \in E(G)$. Then x_3 is adjacent to all the vertices in X and x_1, x_2 are not adjacent to at most one vertex in X. If $\deg x_1$ or $\deg x_2$ is $n - 2$ then $\{x_1, x_2, x_3\}$ is a nc2d-set of G and hence $\gamma_{2nc} \leq 3$. Then $n \leq 5$. If $n = 4$ then G is isomorphic to P_4 or $K_3(1, 0, 0)$. But for these graphs $\gamma_{2nc}(G) + \kappa(G) \neq 2n - 5$. Suppose $n = 5$. Let $X = \{v_1, v_2\}$. Suppose $v_1 v_2 \in E(G)$. If $\deg x_2 = 2$ then G is isomorphic to G_1. If $\deg x_2 = 3$ then for this graph $\gamma_{2nc}(G) + \kappa(G) \neq 2n - 5$. Suppose $v_1 v_2 \notin E(G)$ Then G is isomorphic to G_2 or G_3. Suppose $\deg x_1 = \deg x_2 = n - 3$. If $N(x_1) = N(x_2)$ then there is a vertex $v_1 \in X$ such that v_1 is not adjacent to both x_1 and x_2. Then v_1 is adjacent to all the vertices in X. If $|X| \geq 4$ then $\{v_2, v_3\}$ is a nc2d-set of G and hence $n \leq 4$ which is a contradiction. If $|X| = 3$ then $\{v_1, v_2, v_3\}$ is a nc2d-set of G and hence $n \leq 5$ which is a contradiction.

If $N(x_1) \neq N(x_2)$ then two vertices say v_1 and v_2 such that v_1 is not adjacent to x_1 but adjacent to x_2 and v_2 is not adjacent to x_2 but adjacent to x_1. Then $\{x_1, x_2, x_3\}$ is a nc2d-set of G and hence $n \leq 5$. Then G is isomorphic to C_5 or G_2.

Case 4. $\gamma_{2nc}(G) = n - 3$ and $\kappa(G) = n - 2$.

Then $n-2 \leq \delta(G)$. If $\delta = n-1$ then G is a complete graph which gives a contradiction. Hence $\delta(G) = n-2$. Then G is isomorphic to $K_n - Y$ where Y is a matching in K_n. Then $\gamma_{2nc}(G) \leq 3$. If $\gamma_{2nc}(G) = 3$ then $n = 6$. But $\gamma_{2nc}(K_6 - Y) = 2 \neq n-3$ which is a contradiction. If $\gamma_{2nc}(G) = 2$ then $n = 5$. Hence G is isomorphic to $K_5 - Y$ where Y is any matching in K_5.

Case 5. $\gamma_{2nc}(G) = n-4$ and $\kappa(G) = n-1$.

Then G is a complete graph on n vertices. Since $\gamma_{2nc}(G) = n-4$ we have $n = 6$. Hence G is isomorphic to K_6. The converse is obvious. □

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