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O-ideal Characterization of Normal Almost Distributive Lattices

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Abstract: The class of normal almost distributive lattices is characterized in terms of their O-ideals. Later, existence of the greatest O-ideal contained in a given ideal, is proved. The concept of O-almost distributive lattices is introduced. A necessary and sufficient condition is derived for every generalized Stone almost distributive lattice to become an O-almost distributive lattice.

Keywords: Almost Distributive Lattice(ADL), filter, O-ideal, minimal prime ideal, generalized Stone ADL, O-ADL, normal ADL

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Introduction

In 1981, the notion of Almost Distributed Lattices(ADLs) was first introduced by U.M. Swamy and G.C. Rao [7]. Recently in 2009, the class of normal ADLs was introduced by G.C. Rao and S. Ravikumar [6]. In the paper [4], the authors introduced the concept of O-ideals in an ADL and characterized in terms of minimal prime ideal. It was also observed that the class of O-ideal is not a sublattice of the ideal lattice. In this paper, the main emphasis is given to this feature. A set of equivalent conditions are derived for the class of all O-ideals of an ADL to become

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a sublattice of the ideal lattice, which leads to a characterization of normal ADLs. As a consequence of this result, it is then obtained, the existence of the greatest O-ideal contained in a given ideal. Later, the concept of O-almost distributive lattices is introduced. It is then proved that every O-ADL is a generalized Stone ADL. Finally, a necessary and sufficient condition is derived for every generalized Stone ADL to become an O-ADL.

1 Preliminaries

In this section, we present some definitions and important results taken mostly from [2], [4], [5], [7] and [8] those will be required in the text of the paper.

Definition 1.1. [7] An Almost Distributive Lattice(ADL)with zero is an algebra $(L, \vee, \wedge, 0)$ of type (2,2,0) satisfies the following properties:

1. $(x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z)$
2. $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
3. $(x \vee y) \wedge y = y$
4. $(x \vee y) \wedge x = x$
5. $x \vee (x \wedge y) = x$
6. $0 \wedge x = 0$ for any $x, y, z \in L$

Let X be a non-empty set and $x_0 \in X$ a fixed element. Then for any $x, y \in X$, define $x \vee y = y$ for $x = x_0$, otherwise $x \vee y = x$. Also $x \wedge y = x_0$ for $x = x_0$, otherwise $x \wedge y = y$. Then clearly $(X, \vee, \wedge, x_0)$ is an ADL with x_0 as zero element and is called a discrete ADL. If $(L, \vee, \wedge, 0)$ is an ADL, for any $a, b \in L$, define $a \leq b$ if and only if $a = a \wedge b$ (or equivalently, $a \vee b = b$), then $\leq$ is a partial ordering on L. Throughout this paper, L stands for an ADL $(L, \vee, \wedge, 0)$.

Theorem 1.2. *[7] For any $a, b, c \in L$, we have the following.*

1. $a \vee b = a \Leftrightarrow a \wedge b = b$
2. $a \vee b = b \Leftrightarrow a \wedge b = a$
3. $a \wedge b = b \wedge a$ *whenever* $a \leq b$
4. $\wedge$ *is associative in* L
5. $a \wedge b \wedge c = b \wedge a \wedge c$
6. $(a \vee b) \wedge c = (b \vee a) \wedge c$
7. $a \wedge b = 0 \Leftrightarrow b \wedge a = 0$
8. $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

9. $a \wedge (a \vee b) = a, (a \wedge b) \vee b = b,$ *and* $a \vee (b \wedge a) = a$
10. $a \leq a \vee b$ *and* $a \wedge b \leq b$
11. $a \wedge a = a$ *and* $a \vee a = a$
12. $0 \vee a = a$ *and* $a \wedge 0 = 0$.

An element $m \in L$ is called maximal if it is maximal in the partial ordered set $(L, \leq)$ [7]. That is, for any $x \in L, m \leq x \Rightarrow m = x$.

Theorem 1.3. *[7] For any $m \in L$, the following conditions are equivalent.*

1). m is a maximal element with respect to $\leq$
2). $m \vee x = m$, for all $x \in L$
3). $m \wedge x = x$, for all $x \in L$.

A non-empty subset I of L is called an ideal(filter)[7] of L if $a \vee b \in I (a \wedge b \in I)$ and $a \wedge x \in I (x \vee a \in I)$ whenever $a, b \in I$ and $x \in L$. The set $\mathcal{I}(L)$ of all ideals of L is a complete distributive lattice with the least element $\{0\}$ and the greatest element L under set inclusion in which, for any $I, J \in \mathcal{I}(L)$, $I \cap J$ is the infemum of I, J and the supremum is given by $I \vee J = \{ i \vee j \mid i \in I, j \in J \}$. An ideal I of L is called proper if $I \neq L$. An ideal I of an ADL L is called a direct factor of L if there exists an ideal J of L such that $I \cap J = \{0\}$ and $I \vee J = L$. For any $a \in L$, $(a] = \{ a \wedge x \mid x \in L \}$ is the principal ideal generated by a. Similarly, for any $a \in L$, $[a) = \{ x \vee a \mid x \in L \}$ is the principal filter generated by a. The set $\mathcal{PI}(L)$ of all principal ideals is a sublattice of $\mathcal{I}(L)$. A proper ideal P is said to be prime if for any $x, y \in L$, $x \wedge y \in P \Rightarrow x \in P$ or $y \in P$. A subset P of L is a prime ideal if and only if $L - P$ is a prime filter. A prime ideal P is called a minimal prime ideal[5] if there is no prime ideal Q such that $Q \subset P$. A proper filter M of L is maximal if and only if $L - M$ is a minimal prime ideal.

Theorem 1.4. *[5] A prime ideal P of an ADL L is a minimal prime ideal if and only if to each $x \in P$ there exists $y \notin P$ such that $x \wedge y = 0$.*

For any $A \subseteq L$, $A^* = \{ x \in L \mid a \wedge x = 0 \text{ for all } a \in A \}$ is an ideal of L. We write $(a]^*$ for $\{a\}^*$ and is called an Annulet [3]. Clearly $(0]^* = L$ and $L^* = (0]$.

Lemma 1.5. *[2] For any two ideals I, J of L, we have the following:*

1). If $I \subseteq J$, then $J^* \subseteq I^*$
2). $I \subseteq I^{**}$
3). $I^{***} = I^*$
4). $(I \vee J)^* = I^* \cap J^*$

Definition 1.6. [4] For any filter F of an ADL L, define the set $O(F) = \{ x \in L \mid x \wedge f = 0$ for some $f \in F \} = \bigcup_{x \in F} (x]^*$.

Lemma 1.7. *[4] For any two filters F, G of L, we have the following:*

(a). $O(F)$ is an ideal of L
(b). $F \subseteq G$ implies $O(F) \subseteq O(G)$
(c). $O(F \cap G) = O(F) \cap O(G)$.

An ideal I of an ADL is called an O-ideal [4] if $I = O(F)$, for some filter F of L. An element $x \in L$ is called dense [9] if $(x]^* = (0]$. An ADL L is called a generalized Stone ADL[3] if $(x]^* \vee (x]^{**} = L$ for each $x \in L$. An ADL L is a normal ADL [6] if and only if $(x]^* \vee (y]^* = L$ for all $x, y \in L$ with $x \wedge y = 0$ if and only if $(x]^* \vee (y]^* = (x \wedge y]^*$ for all $x, y \in L$.

2 Characterization of normal ADLs

In this section, some properties of O-ideals are studied. A set of equivalent conditions are established for the class of all O-ideals of an ADL to become a sublattice to the ideal lattice, which leads to a characterization of Normal ADLs.

We first prove some lemmas which we need.

Lemma 2.1. *For any filter F of an ADL L and $x \in L$, we have the following*

(i). $O([x)) = (x]^*$
(ii). $F \cap O(F) \neq \emptyset$ implies that $F = O(F) = L$.

Proof. (i). It is clear that $(x]^* \subseteq O([x))$. Conversely, let $t \in O([x))$. Then $t \wedge a = 0$ for some $a \in [x)$. Hence we get $a \wedge x = x$. Now $t \wedge x = t \wedge a \wedge x = 0$.
(ii). Suppose $x \in F \cap O(F)$. Then we get $x \in F$ and $x \wedge f = 0$ for some $f \in F$. Since $x, f \in F$, we get that $0 = x \wedge f \in F$. Therefore $F = O(F) = L$. □

Lemma 2.2. *Every proper O-ideal is contained in a minimal prime ideal.*

Proof. Let J be a proper O-ideal of L. Then $J = O(F)$ for some filter F of L. Clearly $J \cap F = O(F) \cap F = \emptyset$. Let $\Im = \{G \mid G$ is a filter such that $F \subseteq G$ and $J \cap G = \emptyset\}$. Clearly $F \in \Im$ and $\Im$ satisfies the Zorn's lemma. Let M be a maximal element of $\Im$. We now claim that M is a maximal filter of L. Suppose M_0 is a proper filter of L such that $M \subset M_0$. By the maximality of M and

$F \subseteq M \subset M_0$, we can get $J \cap M_0 \neq \emptyset$. Choose $x \in J \cap M_0$. Then we can get $x \wedge y = 0$ for some $y \in F$. Hence $x \in M_0$ and $y \in F \subseteq M \subset M_0$ implies that $0 = x \wedge y \in M_0$. Which is a contradiction. Thus M is a maximal filter such that $J \cap M = \emptyset$. Therefore $L - M$ is a minimal prime ideal such that $J \subseteq L - M$. $\square$

Let us denote the set of all O-ideals of L by $\mathcal{I}_0(L)$. In [4], it was proved that the intersection of O-ideals is again an O-ideal. But, in general, the join of two O-ideals need not be an O-ideal. It can be seen in the following example.

Example 2.3. Consider the distributive lattice $L = \{0, a, b, c, 1\}$ whose Hasse diagram is given in the figure 2.4.

Consider the ideals $I = \{0, a\}$ and $J = \{0, b\}$.
Clearly $F = \{b, c, 1\}$ and $G = \{a, c, 1\}$ are filters in L.
Now $O(F) = \{0, a\} = I$ and
$O(G) = \{0, b\} = J$. So I, J are O-ideals of L.
But $I \vee J = \{0, a, b, c\}$ is not an O-ideal,
because $c \in L$ and $(c]^* = \{0\}$.
Thus $I \vee J$ is not an O-ideal in L.
Therefore $\mathcal{I}_0(L)$ is not a sublattice of $\mathcal{I}(L)$.

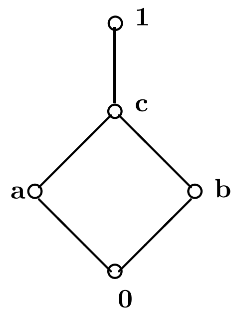

Figure 2.4

However, we have the following.

Theorem 2.4. *The following conditions are equivalent in an ADL L.*

(a). L is normal
(b). For any two filters F, G of L, $F \vee G = L$ implies $O(F) \vee O(G) = L$
(c). For any two filters F, G of L, $O(F) \vee O(G) = O(F \vee G)$
(d). $\mathcal{I}_0(L)$ is a sublattice of $\mathcal{I}(L)$

Proof. $(a) \Rightarrow (b)$: Assume that L is normal. Let F, G be two filters of L such that $F \vee G = L$. Hence we can have $0 = f \wedge g$ for some $f \in F$ and $g \in G$. Since L is normal, $f \in F$ and $g \in G$, we can get that $L = (f]^* \vee (g]^* \subseteq O(F) \vee O(G)$.
$(b) \Rightarrow (c)$: Let F, G be two filters of L. We have always $O(F) \vee O(G) \subseteq O(F \vee G)$. Let $x \in O(F \vee G)$. Then $x \wedge a = 0$ for some $a \in F \vee G$. Now

$$\begin{aligned} a \in F \vee G &\Rightarrow x \wedge (f \wedge g) = 0 \qquad \text{where } f \in F \text{ and } g \in G \\ &\Rightarrow [(x \wedge f) \wedge (x \wedge g)) = [0) \end{aligned}$$

$$\Rightarrow\ [x \wedge f) \vee [x \wedge g) = L$$
$$\Rightarrow\ O([x \wedge f)) \vee O([x \wedge g)) = L$$
$$\Rightarrow\ (x \wedge f]^* \vee (x \wedge g]^* = L$$

Hence $x \in (x \wedge f]^* \vee (x \wedge g]^*$. Thus $x = a \vee b$ where $a \in (x \wedge f]^*$ and $b \in (x \wedge g]^*$. Now

$$\begin{aligned} x &= x \wedge x \\ &= x \wedge (a \vee b) \\ &= (x \wedge a) \vee (x \wedge b) \\ &\in (f]^* \vee (g]^* && \text{since } a \in (x \wedge f]^*,\ \ b \in (x \wedge g]^* \\ &\subseteq O(F) \vee O(G) && \text{since } f \in F \text{ and } g \in G \end{aligned}$$

Hence we get that $O(F \vee G) \subseteq O(F) \vee O(G)$. Therefore $O(F \vee G) = O(F) \vee O(G)$.

$(c) \Rightarrow (d)$: It is obvious.

$(d) \Rightarrow (a)$: Assume that $\mathcal{I}_0(L)$ is a sublattice of $\mathcal{I}(L)$. Let $x, y \in L$ be such that $x \wedge y = 0$. Suppose $(x]^* \vee (y]^* \neq L$. Since $(x]^*, (y]^*$ are O-ideals, by hypothesis we get that $(x]^* \vee (y]^*$ is a proper O-ideal. Hence by Lemma 1.2, there exists a minimal prime ideal P such that $(x]^* \vee (y]^* \subseteq P$. Hence $(x]^* \subseteq P$ and $(y]^* \subseteq P$. Since P is a minimal prime ideal, we get that $x \notin P$ and $y \notin P$. Since P is prime, we get that $0 = x \wedge y \notin P$. Which is a contradiction. Hence we must have $(x]^* \vee (y]^* = L$. Therefore L is normal. □

Corollary 2.5. *Let L be a normal ADL and $\{I_\alpha\}$ an arbitrary family of O-ideals in L. Then $\bigvee_\alpha I_\alpha$ is an O-ideal in L.*

Proof. Let $I_\alpha = O(\vee F_\alpha)$ where F_α is a family of filters of L. Clearly $\vee I_\alpha \subseteq O(\vee F_\alpha)$. Conversely, let $x \in O(\vee F_\alpha)$. Then $x \wedge f = 0$ for some $f \in \vee F_\alpha$. Hence $f = f_1 \wedge f_2 \wedge ..... \wedge f_n$ for some $f_i \in F_{\alpha_i}$. Now

$$\begin{aligned} x \wedge f = 0 &\Rightarrow\ x \wedge f_1 \wedge f_2 \wedge ..... \wedge f_n = 0 \\ &\Rightarrow\ (x \wedge f_1) \wedge (x \wedge f_2) \wedge .....(x \wedge f_n) = 0 \\ &\Rightarrow\ [x \wedge f_1) \vee [x \wedge f_2) \vee ..... \vee [x \wedge f_n) = L \\ &\Rightarrow\ O([x \wedge f_1)) \vee O([x \wedge f_2)) \vee ..... \vee O([x \wedge f_n)) = L \\ &\Rightarrow\ (x \wedge f_1]^* \vee (x \wedge f_2]^* \vee ..... \vee (x \wedge f_n]^* = L \end{aligned}$$

Hence we get $x = a_1 \vee a_2 \vee ..... \vee a_n$ where $a_i \in (x \wedge f_i]^*$. Now $x = x \wedge x = (a_1 \vee a_2 \vee ..... \vee a_n) \wedge x = (a_1 \wedge x) \vee (a_2 \wedge x) \vee ..... \vee (a_n \wedge x) \in (f_1]^* \vee (f_2]^* \vee ..... \vee (f_n]^* \subseteq O(F_{\alpha_1}) \vee O(F_{\alpha_2}) \vee .....O(F_{\alpha_n}) \subseteq \bigvee I_\alpha$. Thus the proof is completed. □

In view of the above theorem, we now obtain the existence of the greatest O-ideal contained in a given ideal of a normal ADL, in the following theorem.

Theorem 2.6. *Let L be a normal ADL. Then for any ideal I which contains an O-ideal K, there exists a largest O-ideal containing K and contained in I.*

Proof. Let I be an arbitrary ideal of L containing an O-ideal K of L. Then consider the set $\Im_K = \{ J \mid J \text{ is an O-ideal such that } K \subseteq J \subseteq I \}$. Clearly $K \in \Im_K$. Let $\{J_i\}_{i\in\Delta}$ be a chain in $\Im_K$. Then clearly $\bigcup J_i$ is an O-ideal and $K \subseteq \bigcup J_i \subseteq I$. So, by Zorn's lemma, $\Im_K$ has a maximal element, say M. We now prove that M is unique. Suppose M_1 and M_2 are two maximal elements of $\Im_K$. Then clearly $K \subseteq M_1 \vee M_2 \subseteq I$. Since L is normal, by Theorem 1.5, we get that $M_1 \vee M_2 \in \Im_K$. Thus we can obtain $M_1 = M_1 \vee M_2 = M_2$. Therefore there is a unique maximal element in $\Im_K$ which is the required largest O-ideal contained in I and containing K. □

If L has dense elements, then it was observed in [4] that $\{0\}$ is an O-ideal. Hence by replacing the arbitrary O-ideal K of the above theorem by the O-ideal $\{0\}$, the following corollary is a direct consequence.

Corollary 2.7. *Let L be a normal ADL with dense elements. Then for any ideal I of L, there exists the greatest O-ideal contained in I.*

Let us denote that I_0 is the greatest O-ideal of L contained in a given ideal I. Then we characterize the elements of this I_0 in the following theorem.

Theorem 2.8. *Let L be a normal ADL with dense elements. For any ideal I*

$$I_0 = \{ x \in L \mid (x]^* \vee I = L \}$$

Proof. It can be easily observed that I_0 is an ideal of L such that $I_0 \subseteq I$. Consider $F = \{ x \in L \mid (x]^{**} \vee I = L \}$. It can be easily observed that F is a filter in L and $I_0 = O(F)$. Let J be an O-ideal of L such that $J \subseteq I$. Since J is an O-ideal, we get $J = O(G)$ for some filter G of L. Let $x \in J$. Then $x \wedge g = 0$ for some $g \in G$. Since L is normal, we get $(x]^* \vee (g]^* = L$. Then

$$L = (x]^* \vee (g]^* \subseteq (x]^* \vee O(G) = (x]^* \vee J \subseteq (x]^* \vee I$$

Hence we get that $x \in I_0$. Therefore I_0 is the greatest O-ideal contained in I. □

3 O-Almost Distributive Lattices

In this section, the concept of an O-Almost Distributive Lattice(simply O-ADL) is introduced. It is proved that the class of all generalized Stone ADLs properly includes the class of all O-ADLs. A necessary and sufficient condition is derived for every generalized Stone ADL to become an O-ADL.

Definition 3.1. An ADL L is called an O-ADL if it satisfies the property.

$$O(F) \vee O(F)^{*} = L \quad \text{for every filter } F \text{ of } L$$

In general, the property $O(F) \vee O(F)^{*} = L$, (for every filter F), need not be hold even in a distributive lattice. It can be observed in the example 2.3. Consider the filter $F = \{b, c, 1\}$ of L. Then $O(G) = \{0, a\}$ and hence $O(G)^{*} = \{0, b\}$. Hence $O(F) \vee O(F)^{*} = \{0, a\} \vee \{0, b\} = \{0, a, b, c\} \neq L$. Therefore L is not an O-ADL. However, an example for an O-ADL is given in the following.

Example 3.2. Let $A = \{0, a\}$ and $B = \{0, b_1, b_2\}$ be two discrete ADLs. Write $L = A \times B = \{(0,0), (0,b_1), (0,b_2), (a,0), (a,b_1), (a,b_2)\}$. Then $(L, \vee, \wedge, 0')$ is an ADL where the zero element is $0' = (0,0)$, under point-wise operations. It can be easily observed that $F_1 = \{(a,b_1), (a,b_2)\}, F_2 = \{(a,0), (a,b_1), (a,b_2)\}$, $F_3 = \{(0,b_1), (0,b_2), (a,b_1), (a,b_2)\}$ are the only filters of L. Now we can get that $O(F_1) = \{(0,0)\}$ and $O(F_1)^{*} = L$. $O(F_2) = \{(0,0), (0,b_1), (0,b_2)\}$ and $O(F_2)^{*} = O(F_3)$. $O(F_3) = \{(0,0), (a,0)\}$ and $O(F_3)^{*} = O(F_2)$. Also observe that $O(F_i) \vee O(F_i)^{*} = L$ for $i = 1, 2, 3$. Hence L is an O-ADL.

Remark. By the definition of an O-ADL, it can be observed that every O-ideal is a direct factor of L. Conversely, let F be a filter of L. Then $O(F)$ is an O-ideal of L. Then there exists an ideal J of L such that $O(F) \cap J = (0]$ and $O(F) \vee J = L$. Now $O(F) \cap J = (0]$ implies that $J \subseteq O(F)^{*}$. Hence $L = O(F) \vee J \subseteq O(F) \vee O(F)^{*}$. Therefore L is an O-ADL.

Theorem 3.3. *Every O-ADL is a generalized Stone ADL.*

Proof. Assume that L is an O-ADL. Let $x \in L$. Clearly $(x]^{*}$ is an O-ideal. Hence by above remark, there exists an ideal J of L such that $(x]^{*} \cap J = (0]$ and $(x]^{*} \vee J = L$. Since $(x]^{*} \cap J = (0]$, we get that $J \subseteq (x]^{**}$. Now we can obtain $L = (x]^{*} \vee J \subseteq (x]^{*} \vee (x]^{**}$. Therefore L is a generalized Stone ADL. □

Since every generalized Stone ADL is a normal ADL[3]the following corollary is a direct consequence of the above theorem.

Corollary 3.4. *Every O-ADL is normal.*

But the converse of above theorem 3.3 is not true. However, we give a sufficient condition for a generalized Stone ADL to become an O-ADL.

Theorem 3.5. *A generalized Stone ADL in which every filter is a principal filter, is an O-ADL.*

Proof. Let L be a generalized Stone ADL in which every filter is a principal filter. Let F be a filter of L. Then $F = [a)$ for some $a \in L$. Now $O(F) \vee O(F)^* = O([a)) \vee O([a))^* = (a]^* \vee (a]^{**} = L$. Therefore L is an O-ADL. □

Moreover, if L has a maximal element, then we derive a necessary and sufficient condition for every generalized Stone ADL to become an O-ADL.

Theorem 3.6. *A generalized Stone ADL with a maximal element m is an O-ADL if and only if every O-ideal is an annulet.*

Proof. Let L be a generalized Stone ADL. Assume that L is an O-ADL. Let I be an O-ADL of L. Then $I = O(F)$ for some filter F of L. Since L is an O-ADL, we get $I \vee I^* = L$. Hence $m = a \vee b$ for some $a \in I$ and $b \in I^*$. Since $b \in I^*$, we get $I \subseteq I^{**} \subseteq (b]^*$. Again, let $c \in (b]^*$. Now $c = m \wedge c = (a \vee b) \wedge c = (a \wedge c) \vee (b \wedge c) = a \wedge c \in I$. Hence $I = (b]^*$. Conversely, assume that each O-ideal is an annulet. Let F be a filter of L. Then $O(F) = (x]^*$ for some $x \in L$. Hence $O(F) \vee O(F)^* = (x]^* \vee (x]^{**} = L$. Therefore L is an O-ADL. □

In the light of the results discussed above, we would like to conclude that the properties of O-ideals provide scope for the further investigations and particularly the nature of primeness of O-ideals may leads to some more results.

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